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### Debt-at-Risk

#### ESM-SUERF-Bruegel Conference: Is Europe Ready for Extreme Events?

Davide Furceri Domenico Giannone Faizaan Kisat W. Raphael Lam

International Monetary Fund

April 3-4, 2025

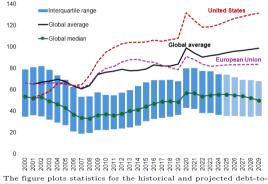
Global debt is elevated and could rise more than anticipated

- High debt reduces fiscal space and raises the risk of sovereign stress.
- Assessing the risks surrounding the debt outlook is thus essential.

#### Public Debt-to-GDP Ratio, 2000-29

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(Percent of GDP)



GDP ratios.

Introduction

Data and Empirical Strategy Results

Source: IMF World Economic Outlook database.

Novel "debt-at-risk" (DaR) framework to quantify the full distribution of risks around debt projections.

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Builds on the "growth-at-risk" methodology (Adrian, Boyarchenko, and Giannone 2019):

- Panel quantile regressions for a sample of 90 countries to construct predicted quantiles of future debt at a forecast horizon of one to five years.
- $\bullet\,$  Estimates fitted to a skewed t-distribution.

Results

• Densities conditional on multiple variables are combined to a single distribution based on the individual factors' predictive power.

Distinctive advantages of DaR:

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- Goes beyond proximate drivers to consider underlying factors—e.g., financial stress.
- Examines their nonlinear effects on the debt distribution.
- Gauges how high debt could rise in an extreme adverse, but plausible, scenario.
- Assesses the relationship between debt-at-risk and fiscal crises.

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Preview	w of results					

- Several financial, political, and economic variables shift the entire future debt distribution, with stronger effects at the right tail.
- Global debt-at-risk is 116 percent of GDP for 2027, nearly 20 percentage points of GDP above WEO projections.
- Debt-at-risk differs across countries and country income groups.
  - Advanced economies: 130 percent of GDP; Emerging markets: 95 percent.
  - Economic and political uncertainty are more important drivers of debt risk for emerging markets; financial factors matter more for advanced economies.
- Debt-at-risk predicts fiscal crises; outperforms other standard indebtedness measures.

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Data					

Data structure:

- Country x year panel.
- Years: 1980-2024, Countries: 90 advanced and emerging and developing economies (>90 percent of global debt)

Key variables:

- Financial variables: Financial Conditions Index (IMF), Financial Stress Index (Ahir et al. 2023), spreads.
- Political variables: World Uncertainty Index (Ahir, Bloom, and Furceri 2022), Reported Social Unrest Index (Barrett et al. 2022).
- Economic variables: Debt-to-GDP, primary balance, real GDP growth, inflation (WEO database).

## Empirical framework–Quantile regression specification

Estimate the following panel location-scale model (Machado and Santos Silva 2019):

$$d_{i,t+h} = \alpha_i + X'_{i,t}\beta + (\delta_i + X'_{i,t}\gamma)\varepsilon_{i,t+h}$$
(1)

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- $d_{i,t+h}$ : h year-ahead debt-to-GDP ratio (h :1 to 5 years), of country i in year t.
- $\alpha_i$  and  $\delta_i$ : Country fixed effects.

Data and Empirical Strategy

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- $X_{i,t}$ : Vector of predictors (initial debt included in all specifications).
- $\beta$ : Location parameter–"shift" in the entire distribution of future debt as a regressor moves.
- $\gamma$ : Scale parameter–captures whether this shift differs across quantiles.

The  $\tau$ -th quantile of future debt,  $Q_d(\tau)$  is given by:

$$Q_{d_{i,t+h}}(\tau|X_{i,t}) = (\alpha_i + \delta_i q(\tau)) + X'_{i,t}\beta + X'_{i,t}\gamma q(\tau)$$
<sup>(2)</sup>

where  $q(\tau) = F_{\varepsilon}^{-1}(\tau)$ . Quantile regression coefficient for predictor x:  $\beta_x + \gamma_x q(\tau)$ .

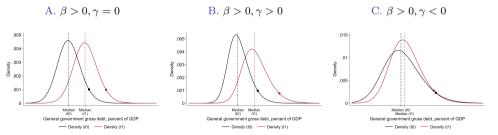
## Effects of a conditioning variable on the future debt distribution depends on the location and scale parameters

Data and Empirical Strategy Results

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Impact of Conditioning Variable on Density Based on Location ( $\beta$ ) and Scale ( $\gamma$ ) Coefficients

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The figures plot illustrative changes in the predicted conditional debt densities when the conditioning variable increases by one standard deviation. t0 refers to period zero, and t1 refers to period one (after the change in the conditioning variable). In Panel A,  $\beta$  is positive but  $\gamma$  is zero; In Panel B, both  $\beta$  and  $\gamma$  are positive (same signs); In Panel C,  $\beta$  is positive while  $\gamma$  is negative (opposite sign).



Fitting densities: Details

- Quantiles re-centered such that predicted median conditional on initial debt matches WEO 2025-30 projection.
- Predicted quantiles fitted to a skewed *t*-distribution (Azzalini and Capitanio 2003). Combining densities:
  - For a particular country, year, and forecast horizon, we pool densities using a weighted sum of the densities based on individual predictors m:

$$\hat{f}_{i,t+h}^{pooled}(d) = \sum_{m} \eta_{i,h}^{m} \hat{f}_{i,t+h}^{m}(d)$$
 (3)

• Weights  $\eta_{i,h}^m$  sum to one and maximize out-of-sample predictive accuracy of the combined distribution (Crump et al. 2023). Details

Quantiles aggregated to global or country-group level using GDP weights Details. Global sample has 47 countries List.

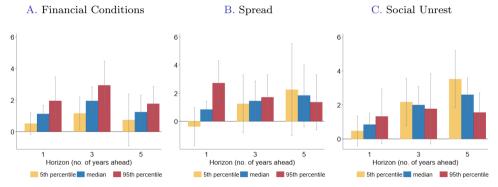
Adverse financial and political developments are associated with higher debt-at-risk

Results

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Quantile Regression Results: Forward Debt-to-GDP Ratio and Financial and Political Variables

Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion

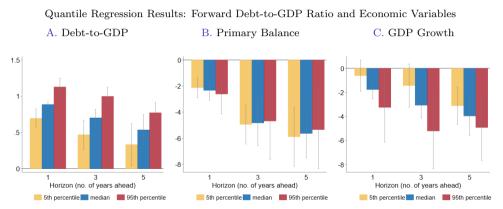


The figure displays the estimated quantile regression coefficients for 5th, 50th, and 95th percentile based on panel quantile regressions of the future debt-to-GDP ratio on selected financial, political, and economic variables (Equation (1)). Bars denote estimated coefficients, and the whisker in each bar shows the associated 90 percent confidence interval. The coefficients refer to the percentage point change in the government debt-to-GDP ratio when the explanatory variable changes by one unit. All explanatory variables (except for initial debt) are standardized to have a mean of zero and a standard deviation of one to ensure comparability across coefficients. Standard errors are clustered at the country level.

# Economic factors have persistent and asymmetric effects on the debt distribution

Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion

Results



The figure displays the estimated quantile regression coefficients for 5th, 50th, and 95th percentile based on panel quantile regressions of the future debt-to-GDP ratio on selected financial, political, and economic variables (Equation (1)). Bars denote estimated coefficients, and the whisker in each bar shows the associated 90 percent confidence interval. The coefficients refer to the percentage point change in the government debt-to-GDP ratio when the explanatory variable changes by one unit. All explanatory variables (except for initial debt) are standardized to have a mean of zero and a standard deviation of one to ensure comparability across coefficients. Standard errors are clustered at the country level.

## Introduction Data and Empirical Strategy Results October 2000 Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion of Significant and asymmetric effects of conditioning factors

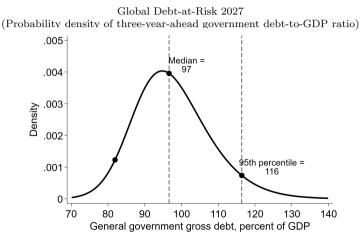
#### Location-Scale Coefficients: Forward Debt-to-GDP Ratio vs. Financial, Political, and Economic Variables

	:	1	Horizon (no. of	. ,	5	5
	Location (1)	Scale (2)	Location (3)	Scale (4)	Location (5)	Scale (6)
Panel A: Financial Var	iables					
Financial Conditions Spread	$1.164^{***}$ $0.960^{***}$	$0.299 \\ 0.704^{**}$	$1.983^{***}$ $1.459^{**}$	$0.443^{**}$ 0.119	$1.236^{**}$ 1.832	$0.279 \\ -0.230$
Panel B: Political Varia	ables					
Social Unrest	$0.867^{**}$	0.172	1.988***	-0.095	$2.587^{***}$	-0.507*
Panel C: Economic Var	riables					
Debt-to-GDP Primary Balance GDP Growth	0.899*** -2.341*** -1.849***	0.090*** -0.091 -0.524	0.717*** -4.821*** -3.186***	$0.125^{***}$ 0.061 $-0.912^{*}$	0.541*** -5.626*** -3.983***	$0.111^{***}$ 0.137 -0.463

The table displays the estimated location ( $\beta$ ) and scale ( $\gamma$ ) coefficients based on panel quantile regressions of the future debt-to-GDP ratio on selected financial, political, and economic variables (Equation (1)). All explanatory variables (except for initial debt) are standardized to have a mean of zero and a standard deviation of one to ensure comparability across coefficients. Standard errors (reported in parentheses) are clustered at the country level. \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5% and 10% levels, respectively. Global debt-at-risk for 2027 is estimated at about 116 percent of GDP, nearly 20 percentage points higher than projections

Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion

Results

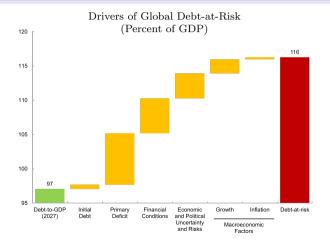


The figure plots the predicted density of the three-year-ahead global debt-to-GDP ratio. The probability density functions are estimated using panel quantile regressions of the debt-to-GDP ratio on various political, economic, and financial variables. The global sample comprises 47 countries for which data on the conditioning variables are available. The quantile estimates are fitted to a skewed t-distribution. Dots indicate the predicted 5th, 50th (median), and 95th quantile of the debt-to-GDP ratio.

Primary deficits and tight financial conditions are the main factors contributing to upside risks to global debt

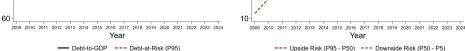
Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion

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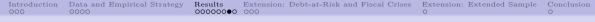


The figure plots the contributions from the conditioning variables used for the debt-at-risk model to the estimated level of global debt-at-risk. The green bar denotes the baseline debt projection for 2027 from the World Economic Outlook database. Yellow bars refer to contribution from the conditioning variables. The red bar indicates the value of the debt-at-risk.

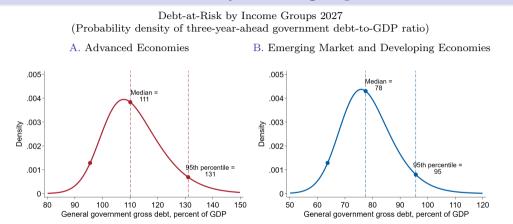
Results Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion 000000000 Global debt-at-risk has risen since 2009, and upside risks are consistently higher than downside risks Evolution of Global Debt and Debt Risks (Percent of GDP) A. Debt-to-GDP and Debt-at-Risk B. Upside and Downside Risks 140 25 120 20 100 15 80



The figures plot the evolution of global GDP weighted debt-to-GDP and debt-at-risk (Panel A), and the evolution of upside and downside debt risks (Panel B) at a three-year forecast horizon. Debt-at-risk is defined as the predicted 95th quantile (P95) of the combined distribution. Upside risks are calculated as the difference between the predicted 95th quantile of the combined distribution and the predicted 50th quantile (median) of the distribution conditional on initial debt (P95 - P50). Downside risks are the difference between the predicted median conditional on initial debt and the predicted 5th quantile of the combined distribution (P50 - P5).



#### Debt-at-risk differs across country income groups...



The figure plots the predicted density of the three-year-ahead global debt-to-GDP ratio for Advanced Economies (Panel A) and Emerging Market and Developing Economies (Panel B). The probability density functions are estimated using panel quantile regressions of the debt-to-GDP ratio on various political, economic, and financial variables. The global sample comprises 47 countries for which data on the conditioning variables are available. The quantile estimates are fitted to a skewed t-distribution. Dots indicate the predicted 5th, foth (median), and 95th quantile of the debt-to-GDP ratio.

ta and Empirical Strategy Results Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion o o

## ... with large primary deficits driving debt risks for AEs and economic and political uncertainty more important for EMDEs

#### Drivers of Global Debt-at-Risk by Income Groups (Percent of GDP)

A. Advanced Economies

135 131 95 130 95 125 90 120 81 115 80 110 76 Debt-to-GDF Initia Priman Financial Economic Groudi Inflation Debt-at-risk Debt-to-GDP Initial Primary Financial Economic Groud Inflation Debt-at-risk (2027)Debt Deficit Conditions and Political (2027)Debt Deficit Conditions and Politica Uncertainty Uncertaint Macroeconomic Massassassi and Rieke Factors and Rieke Factors

B. Emerging Market and Developing Economies

The figure plots the contributions from the conditioning variables used for the debt-at-risk model to the estimated level of debtat-risk for Advanced Economies (Panel A) and Emerging Market and Developing Economies (Panel B). The green bar denotes the baseline debt projection for 2027 from the World Economic Outlook database. Yellow bars refer to contribution from the conditioning variables. The red bar indicates the value of the debt-at-risk.

#### Results Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion ..... Debt-at-Risk and fiscal crises

- Do the debt-at-risk measures help predict fiscal crises?
- If so, how well do they perform relative to other economic variables?

To address these, we:

• Construct a binary fiscal crisis variable (Moreno Badia et al. 2022). (Details)



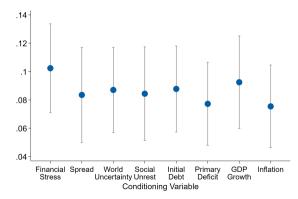
- Correlate crisis variable with debt-at-risk measure:
  - Logit model
  - Bayesian Model Averaging (BMA) model
  - Random forest machine learning model

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Debt-at-Risk correlates strongly and positively with a future fiscal crisis.

Logistic Regression Coefficients: Fiscal Crisis vs. One-Year Ahead Debt-at-Risk



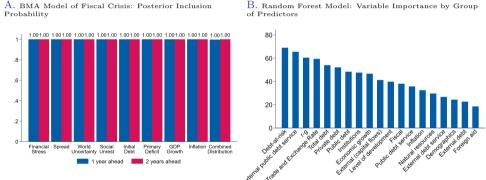
The figure shows estimated coefficients from a panel logit regression of a fiscal crisis indicator against debt-at-risk. Each point denotes the coefficient from a separate regression. The independent variable is the difference between the predicted 95th quantile of one-year-ahead debt-to-GDP and the 50th quantile conditional on the variables displayed on the horizontal axis. Whiskers show the 90 percent confidence intervals.

2. How well do the measures perform in predicting crises relative to other economic variables?

Debt-at-risk is one of the most robust metrics in predicting a fiscal crisis.

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Results



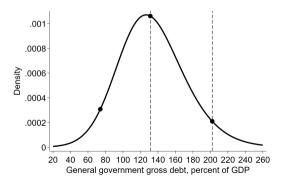
Panel A shows the estimated posterior inclusion probability (PIP) for the interaction of (P95-P50) with P95 from a Bayesian model averaging (BMA) model of a fiscal crisis indicator against debt@risk and other macroeconomic variables. Panel B displays (grouped) average variable importances from a random forest model used to predict a fiscal crisis. The independent variables are similar to those used for the BMA. Higher values indicate that a variable has a higher predictive power.

Extension: Debt-at-Risk and Fiscal Crises Extension: Extended Sample Conclusion



• Construct measure of debt-at-risk for based on more widely available economic variables. Sample extended to 171 countries, including many low-income economies.

Debt-at-Risk 2027 for Selected Low Income Developing Country in Extended Sample (Probability density of three-year-ahead government debt-to-GDP ratio)



The figure plots the predicted density of the three-year-ahead global debt-to-GDP ratio for a selected highly indebted low income developing economy in the extended sample. The economy has coverage for the economic variables but does not have any data for the considered financial and political variables.

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#### Conclusion and policy implications

Conclusion:

- Debt risks are elevated and titled to the upside.
- Global debt-at-risk in 2026 is estimated at around 116 percent of GDP, with significant heterogeneity across countries.
- Debt-at-risk is the most robust predictor of fiscal crises.

Policy implications:

- Policymakers can use the measure to quantify the size of debt risks in a severely adverse scenario; compare risks over time and across countries.
- Debt-at-risk could be used as an early-warning tool to monitor fiscal crises.

## Additional Slides

#### Additional Slides 0000000

### Fitting densities to a skewed t-distribution

- Distribution depends on four parameters: the location  $\mu$ , scale  $\sigma$ , fatness  $\nu$ , and the shape  $\alpha$ .
- Choose parameters to fit predicted quantiles to the quantile function of the distribution:

$$\{\hat{\mu}_{i,t+h}^{m}, \hat{\sigma}_{i,t+h}^{m}, \hat{\alpha}_{i,t+h}^{m}, \hat{\nu}_{i,t+h}^{m}\} = \underset{\mu,\sigma,\alpha,\nu}{\operatorname{argmin}} \sum_{\tau} \left(\hat{Q}_{d_{i,t+h}}^{m}(\tau) - F^{-1}(\tau;\mu,\sigma,\alpha,\nu)\right)^{2}$$
(4)

where:

- $\hat{Q}^m_{d_{i,t+h}}(\tau)$  is predicted quantile from quantile regression (2).
- $F^{-1}(\tau;\mu,\sigma,\alpha,\nu)$  is the quantile function of the skewed *t*-distribution.
- $\tau \in \{5, 25, 75, 95\}$

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## Weighting methodology to combine conditional distributions

Two steps:

- Compute out-of-sample predictive densities conditional on each explanatory variable m using data from the *prior* 20 years (from 2005 onwards).
  - E.g.,  $\hat{p}_{2007|2005}^m(d)$  is density for 2007 conditional on information from 1986 to 2005.
- Weights are the values (positive and summing to one) that maximize these probability scores across all years:

$$\eta_{i,h}^{1}, \dots, \eta_{i,h}^{M} = \operatorname{argmax} \sum_{t=2005+h}^{2023} \sum_{m=1}^{M} \eta_{i,h}^{m} \hat{p}_{T+h|T}^{m}(d)$$
(5)

s.t. 
$$\eta_{i,h} > 0; \sum_{m=1}^{M} \eta_{i,h}^{m} = 1$$

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Additional Slides

### Aggregating to global levels or country groups

• For each model m, approximate the quantile of the global distribution with the weighted average of the country-level quantiles:

$$\hat{Q}^{m}_{d_{global,t+h}}(\tau) = \sum_{i=1}^{I} \omega_{i,t} \hat{Q}^{m}_{d_{i,t+h}}(\tau)$$
(6)

- $\omega_{i,t}$  is country *i*'s nominal GDP share.
- Re-center quantiles around WEO projections.
- Fit aggregate quantiles to skewed t-distribution to obtain global density  $\hat{f}_{alobal,t+h}^m(d)$
- Pool densities by combining model-specific densities; global weights are the GDP-weighted average of the country-specific weights:

$$\hat{f}_{global,t+h}^{poooled}(d) = \sum_{m=1}^{M} \omega_{global,h}^{m} \hat{f}_{global,t+h}^{m}(d)$$
(7)

where  $\omega_{global,h}^m = \sum_{i=1}^I \omega_{i,t} \eta_{i,h}^m$ .

## Country coverage for global debt distribution

Advanced Economies	Emerging Market and Developing Economies
Australia	Brazil
Austria	Bulgaria
Belgium	Chile
Canada	China
Denmark	Colombia
Finland	Hungary
France	India
Germany	Indonesia
Greece	Kenya
Hong Kong SAR	Malaysia
Ireland	Mexico
Israel	Morocco
Italy	Nigeria
Japan	Pakistan
Korea	Peru
Netherlands	Philippines
New Zealand	Romania
Norway	Russia
Portugal	South Africa
Spain	Tanzania
Sweden	Thailand
Switzerland	Vietnam
United Kingdom	Zambia
United States	

## Defining a fiscal crisis

Fiscal crisis occurs if any one of four criteria are met:

- A credit event (default, restructuring, or rescheduling).
- **②** Exceptionally large official financing from the IMF or European Union.
- **③** Implicit default on domestic debt (high inflation, increase in domestic arrears).
- Loss of market confidence (spike in spreads, loss of market access).

#### Back

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