

Housing and the secular decline in real interest rates

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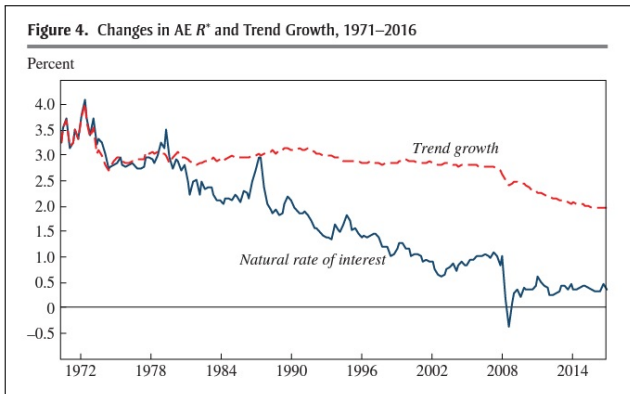
Oesterreichische Nationalbank

“Equilibrium Real Interest Rates — concepts, current and future drivers: New insights and policy implications”,
SUERF/OeNB Workshop,
Vienna, 7th December 2023

*The content of these slides reflects the views of the authors and not necessarily those of the OeNB.

The decline in real long-run interest rates

- Laubach/Williams (2003): Decrease for the United States since 1980: around 3%.
- Rachel/Summers(2019): Decrease for advanced economies since 1970: around 3%.



Explanations for the decline in real interest rates

- **Demographic aging** increased the need for old age provision (Eggertson et al., 2019; Auclert et al., 2021)
- Increasing **income inequality** and the “**saving glut of the rich**” (Mian et al., 2020)
- **Global saving glut** and **safe assets shortage** (Bernanke, 2005; Caballero et al., 2017)
- **Calibrated models** confirm the importance of these factors:
 - **Mankiw (2022)**: A simple Solow model with only s and $g + n$
 - **Rachel/Summers (2019)**: A Blanchard/Yaari/Gertler model
 - **Platzer/Peruffo (2022)**: A large-scale model

This paper

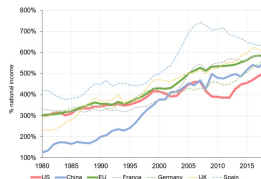
- Study a model that includes an **asset in fixed supply** (land/housing).
- The model (based on Piketty, 2011) is **stylized** in order to provide intuition and (under certain conditions) **closed-form solutions**: deterministic, real-term, closed economy, focus on steady-state comparisons.
- **Main questions**: Does the existence of housing ...
 - **dampen the fall in interest rates** (by absorbing excess savings)?
 - change the **relative importance** of the different channels?
 - help to explain **other stylized facts**?

Other important long-run trends

- Private wealth-income ratios:

300% (1980) → 540% (2018)

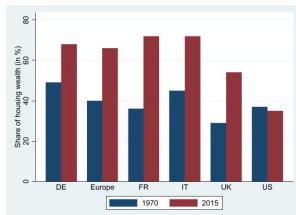
Figure 1: Household Wealth-National Income Ratios, 1980-2018



- Share of housing wealth:

36% (1970) → 53% (2015)

- Other: Inheritance flows, saving rates, volume of mortgages, inequality, ...



Benchmark model

- I consider a structure with **four groups** (r, om, oo, w):
 - Renters
 - Owner-occupiers with mortgage
 - Outright owners (without a mortgage)
 - Top 1% (outright owners, higher bequest motive)
 - Model assumptions:
 - Owners with mortgages continuously refinance their purchases (no transaction costs)
 - Outright owners inherit a certain house and pass it on to their children who do the same....
- Short-cut for:
- Houses that people are not *allowed* to sell (trusts etc.)
 - Houses that people are not *willing* to sell ("old family property" etc.)
 - Sluggishness over the lifecycle ("aging in place" etc.)

Equilibrium interest rate

- **Asset supply** (=Wealth demand):
 - Physical capital K_t
 - Houses \bar{H}_t ($= \bar{H}_t^r + \bar{H}_t^{om} + \bar{H}_t^{oo} + \bar{H}_t^w$)
 - Government bonds D_t
- **Asset demand** (=Wealth supply):
 - By households with a **life-cycle** and a **bequest** motive
- Write the **wealth-to-income ratio** as $\beta \equiv \frac{\text{Wealth}_t}{\text{NDP}_t}$
(and $\beta_Z = \frac{Z_t}{\text{NDP}_t}$ for asset Z_t) ◀ National accounting
- The **equilibrium interest rate** r^* solves:

$$\underbrace{\beta = \beta_K + \beta_{Hr} + \beta_{Hom} + \beta_D}_{\text{Wealth Demand}} = \underbrace{\tilde{\beta}}_{\text{Wealth Supply}}$$

Demography

- Continuous-time OLG model. Individuals ...
 - become adults at age A ,
 - are employed until retirement at age R ,
 - die at age D ,
 - receive a bequest at age I (with $A \leq I \leq R$),
 - receive a pension with net replacement rate ρ after retirement.
- Each cohort born in time x has a size $N^x = N^0 e^{nx}$ and includes a continuum $i \in [0, 1]$ of individuals.

Warm-glow model

Lifetime utility $V^j(A)$ for group $j \in \{r, om, oo, w\}$ consists of:

- Intratemporal function:

$$u^j(a) = \frac{(\eta^j h^j(a))^\gamma (c^j(a))^{1-\gamma}}{(\gamma)^\gamma (1-\gamma)^{1-\gamma}} \rightarrow \text{determines } P_{st}^r \text{ and } P_{st}^o$$

- Intertemporal function:

$$U^j(A) = \left\{ \frac{\int_A^D e^{-\theta(a-A)} (u^j(a))^{1-\sigma} da}{\int_A^D e^{-\theta(a-A)} da} \right\}^{\frac{1}{1-\sigma}}$$

- Intergenerational function:

$$V^j(A) = (1 - s_B^j) \log(U^j(A)) + s_B^j \log(w^j(D))$$

- Determines $\tilde{\beta}$ depending on two crucial savings motives:
 - A life-cycle motive (for net replacement rate $\rho < 1$):
 - A bequest motive (for $s_B^j > 0$).

Production

- Output of “normal” (non-housing) goods and services:

$$Y_{Nt} = K_t^\alpha (\mathcal{A}_t L_t)^{1-\alpha}$$

- Productivity \mathcal{A}_t grows at rate g , labor supply L_t at rate n .
- Factor markets are competitive.
- The net return on capital:

$$r_{kt} = \alpha \frac{Y_{Nt}}{K_t} - \delta_k$$

- The capital-to-income ratio in steady state (with $r_{kt} = r_k$):

$$\beta_K^N \equiv \frac{K_t}{Y_{Nt}} = \frac{\alpha}{r_k + \delta_k}$$

Housing 1

- Housing supply: $\bar{H}_t = \bar{H}_t^r + \bar{H}_t^o$
 - \bar{H}_t^r ... rented stock, $\bar{H}_t^o = \bar{H}_t^{om} + \bar{H}_t^{oo} + \bar{H}_t^w$... owned stock
 - The housing stocks grow at rate n .
- Rental housing:
 - The rental housing-wealth-to-income ratio:

$$\beta_{Hr}^N \equiv \frac{P_{ht}^r \bar{H}_t^r}{Y_{Nt}} = \frac{\frac{P_{st}^r \bar{H}_t^r}{Y_{Nt}}}{r_h + \delta_h - g}$$

- P_{st}^r : rent (*service price*), P_{ht}^r : *purchasing price*
- r_{ht} : Rate of return on investments into rental housing:

$$r_{ht} = \frac{P_{st}^r}{P_{ht}^r} - \delta_h + \frac{\dot{P}_{ht}^r}{P_{ht}^r} \rightarrow P_{ht}^r = \frac{P_{st}^r}{r_{ht} + \delta_h - \frac{\dot{P}_{ht}^r}{P_{ht}^r}}$$

- In the **steady state** $\frac{\dot{P}_{ht}^r}{P_{ht}^r} = g$ and $r_{ht} = r_h$.

Housing 2

- Owned housing:
 - The owned housing-wealth-to-income ratio:

$$\beta_{Ho}^N \equiv \frac{P_{ht}^o \bar{H}_t^o}{Y_{Nt}} = \frac{\frac{P_{st}^o \bar{H}_t^o}{Y_{Nt}}}{r_m + \delta_h - g}$$

- P_{st}^o : imputed rent (shadow service price), P_{ht}^o : purchasing price
- Assumption: All home purchases are fully financed by mortgages at the rate r_{mt} and there is continuous re-financing (no transaction costs).
- Imputed rent: $P_{st}^o = \left(r_{mt} + \delta_h - \frac{\dot{P}_{ht}^o}{P_{ht}^o} \right) P_{ht}^o$.

Summary: Equilibrium interest rate

- Equilibrium condition:

$$\underbrace{\beta = \beta_K + \beta_{Hr} + \beta_{Hom} + \beta_D}_{\text{Wealth Demand}} = \underbrace{\tilde{\beta}}_{\text{Wealth Supply}}$$

- Wealth Demand:

$$\beta_K = \frac{\alpha}{r_k + \delta_k}, \beta_D \text{ (public debt, assumed as given)}$$

$$\beta_{Hr} = \frac{\frac{P_{st}^r \bar{H}_t^r}{Y_{Nt}}}{r_h + \delta_h - g}, \beta_{Ho} = \frac{\frac{P_{st}^o \bar{H}_t^o}{Y_{Nt}}}{r_m + \delta_h - g}$$

- Equilibrium interest rates:

$$r^* = \frac{1}{\beta N} (\beta_K r_k + \beta_{Hr} r_h + \beta_{Ho} r_m + \beta_D r_d)$$

$$r_h = r_k - \xi_h, \quad r_m = r_k - \xi_m, \quad r_d = r_k - \xi_d$$

Calibration

- Focus on **steady-state comparisons** between an “initial” situation (around 1980) and a “current situation” (around 2018).
- The values refer to the group of **advanced countries**
- Demographic and economic parameters

Initial: $g = 3.0\%$, $n = 1.0\%$, $D = 75$, $R = 65$, $\rho = 70\%$, $\beta_D^N = 20\%$

Current: $g = 1.8\%$, $n = 0.5\%$, $D = 82$, $R = 63$, $\rho = 60\%$, $\beta_D^N = 70\%$

- Bequest motive

Initial: s_B and $s_B^{top1\%}$ such that $\beta = 350\%$ and $\frac{\text{Wealth of top } 1\%_t}{\text{Total Wealth}} = 28\%$

Current: $s_B^{top1\%}$ changes such that $\frac{\text{Wealth of top } 1\%_t}{\text{Total Wealth}} = 35\%$

- Risk discounts

Initial: $\xi_h = 0\%$, $\xi_m = 2\%$, $\xi_d = 5\%$, Current: $\xi_m = 3\%$

- Population shares

Renters: $50\% \rightarrow 40\%$, Mortgage owners: $25\% \rightarrow 35\%$

Outright owners: Pop. share constant, share of owned houses \uparrow

Numerical results

Case	r	β	β_K	$\frac{\beta_H}{\beta}$
	Baseline model (4 groups)			
Initial	9.6%	350%	168%	46%
Current	5.7%	599%	204%	54%

Numerical results

Case	r	β	β_K	$\frac{\beta_H}{\beta}$
	Baseline model (4 groups)			
Initial	9.6%	350%	168%	46%
Current	5.7%	599%	204%	54%
	No outright owners (3 groups)			
Initial	9.4%	350%	168%	46%
Current	6.5%	480%	196%	44%

Numerical results

Case	r	β	β_K	$\frac{\beta_H}{\beta}$
	Baseline model (4 groups)			
Initial	9.6%	350%	168%	46%
Current	5.7%	599%	204%	54%
	No outright owners (3 groups)			
Initial	9.4%	350%	168%	46%
Current	6.5%	480%	196%	44%
	No owners (2 groups)			
Initial	9.3%	350%	175%	44%
Current	6.2%	476%	212%	39%

Numerical results

Case	r	β	β_K	$\frac{\beta_H}{\beta}$
	Baseline model (4 groups)			
Initial	9.6%	350%	168%	46%
Current	5.7%	599%	204%	54%
	No outright owners (3 groups)			
Initial	9.4%	350%	168%	46%
Current	6.5%	480%	196%	44%
	No owners (2 groups)			
Initial	9.3%	350%	175%	44%
Current	6.2%	476%	212%	39%
	No housing ($\gamma = 0$, 2 groups)			
Initial	7.7%	249%	225%	0%
Current	4.0%	370%	280%	0%

Results 1

- Decrease in r by 3.9 pp (from 9.6% to 5.7%)
- Increase in β by 250 pp (from 350% to 599%)
- Increase in the β_H/β by 8 pp (from 46% to 54%)
- Results are broadly in line with the observed data.
 - $\Delta r \approx 3$ pp
 - β from 300%-350% (1970-80) to 500%-550% (2015-18)
 - β_H/β from 36% (1970) to 53% (2015)
 - Sources: Piketty/Zucman, 2014; Alvaredo et al., 2018; Bauluz et al., 2022
- Most of the increase in β is due to housing wealth (only moderate increase in β_K).
- Comparison to the case without (outright) owners:
 - Without (outright) owners the model implies a *reduction in the share of housing wealth*. ◀ Intuition
- Comparison to the case without housing:
 - Existence of housing increases r and β (even though $\beta_K \downarrow$).

Results 2

Case	r	$r_k = r_h$	r_m	r_d	β	$\frac{\beta_H}{\beta}$
Initial	9.6%	10.3%	8.3%	5.3%	350%	46%
Current	5.7%	7.2%	4.2%	2.2%	599%	54%

- Interest rates:
 - Mortgage rate r_m : 8.3% \rightarrow 4.2%
 - Interest rate on safe assets: 7.3% \rightarrow 3.3%
 - Interest rate on gov. bonds: 5.3% \rightarrow 2.2%
 - Return on housing (rented & owner-occupied): 9.2% \rightarrow 4.8%

[◀ See](#)
- Mortgages-to-GDP ratio increases from 36% to 74% (in the data from 25% to 65% (Jordà et al., 2016)).
- Inheritance flow increases from 6.6% to 9.2% (in the data from around 6% to around 11%). [◀ See](#)

Comparison to the literature

Decomposition of the decline in the real interest rate in Rachel/Summers (2019), Platzer/Peruffo (2022) and this paper.

Variable	RS '19	PP '22	This paper (4 Groups)
TFP growth (g)	-1.8	-1.00	-1.12
Pop. growth (n)	-0.6	-0.25	-0.39
Longer retirement (D)	-1.1	-0.46	-0.78
Length of working life (R)	-0.1	–	-0.07
Replacement rate (ρ)	–	–	-0.13
Inequality (s_B^w and d_y^w)	-0.7	-0.70	-1.90
Public Debt (β_D^N)	+3.6	+0.31	+0.29
Interactions	-1.1	-0.06	0.12
Other factors	–	0.00	0.03
Total	-1.8	-2.16	-3.95

Additional implications

- Positive correlation of **outright ownership rates** with **aggregate wealth** and with **housing wealth share**. [◀ See](#)
- The evidence in Fagereng et al. (2019) about **“capital gains savers”** is also compatible with the assumption of sticky outright owners. [◀ See](#)
- The model offers an explanation for a divergent trend in the rates of **gross and net savings** [◀ See](#)

Conclusions

- A **model** that includes land/housing is compatible with a decline in the interest rate due to aging, increasing inequality and a slowdown in productivity growth.
- In order to also explain the rise in the housing share it is necessary to introduce **additional elements** like the existence of **owner occupiers**.
- The **full model** is not only (broadly) in line with the developments of r , β and β_H/β but also with **other important magnitudes** like: the inheritance flows, the volume of mortgages, the importance of capital gain savers.

Appendix

National accounting

$$\begin{aligned}
 NDP_t = & \\
 & \underbrace{Y_{Nt} + P_{st}^r \bar{H}_t^r + P_{st}^o \bar{H}_t^o}_{GDP_t^{NA}} + \underbrace{\dot{P}_{ht}^r \bar{H}_t^r + \dot{P}_{ht}^o \bar{H}_t^o}_{\text{Capital Gains}} - \underbrace{\delta_k K_t - \delta_h (P_{ht}^r \bar{H}_t^r + P_{ht}^o \bar{H}_t^o)}_{\text{Depreciations}} \\
 & \underbrace{\hspace{10em}}_{GDP_t = \text{Haig-Simmons national income}}
 \end{aligned}$$

- The **Haig-Simmons definition of national income** is the theory-consistent concept. Its use has been suggested, e.g., by Robbins (2018) and Fagereng et al. (2019).
- Note that empirically often $\frac{NDP_t}{Y_{Nt}} \approx 1 \rightarrow \boxed{\beta_t \approx \beta_t^N}$.
- Since: $\beta_t \equiv \frac{\text{Wealth}_t}{NDP_t} = \frac{\text{Wealth}_t}{Y_{Nt}} \frac{Y_{Nt}}{NDP_t} = \beta_t^N \frac{Y_{Nt}}{NDP_t} \approx \beta_t^N$.

Rates of returns on housing vs. equity

Country	Full sample		Post-1950		Post-1980	
	Equity	Housing	Equity	Housing	Equity	Housing
Average, unweighted	6.67	7.26	8.30	7.47	10.78	6.43
Average, weighted	7.12	6.72	8.19	6.40	9.08	5.50

Source: Jordà et al., 2019

- Depending on the sample a risk discount **between 1% and 4%** seems reasonable (average between ξ_h and ξ_m).
- Eichholtz et al. (2021) and Chambers et al. (2021) find lower (risk-adjusted) returns to rental housing for Amsterdam, Paris and Oxbridge colleges than Jordà et al. (2019).

Intuition for the behavior of the housing share 1

Without outright owners:

$$\frac{\beta_H}{\beta_K} = \frac{\frac{1}{Y_{Nt}} \left(\frac{P_{st}^r \bar{H}_t^r}{r_h + \delta_h - g} + \frac{P_{st}^o \bar{H}_t^{om}}{r_m + \delta_h - g} \right)}{\frac{\alpha}{r_k + \delta_k}}.$$

Simple example:

- Assumptions:

- $P_{st}^r \bar{H}_t^r = \gamma Y_{Lt}^r$, and $P_{st}^o \bar{H}_t^{om} = \gamma Y_{Lt}^o$
- $r_k = r_h = r_m = g + n$, $\delta_k = \delta_h = 0$

- $\frac{\beta_H}{\beta_K} = \frac{\gamma(1-\alpha)}{\alpha} \frac{g+n}{n}$

- $\frac{\partial(\beta_H^N/\beta_K^N)}{\partial t} = \frac{\gamma(1-\alpha)}{\alpha} \frac{g}{n} \left(\frac{\dot{g}}{g} - \frac{\dot{n}}{n} \right)$

Intuition for the behavior of the housing share 2

With outright owners:

$$\frac{\beta_H}{\beta_K} = \frac{\gamma(1-\alpha)}{\alpha} \left[\kappa_N^r \frac{r_k + \delta_k}{r_h + \delta_h - g} + \left(1 - \kappa_N^r + \frac{\kappa_H^{od} - \kappa_N^{od}}{1 - \kappa_H^{od}} \right) \frac{r_k + \delta_k}{r_m + \delta_h - g} \right],$$

$\kappa_N^j \dots$ population size of group j , $\kappa_H^j \dots$ size of the housing stock.

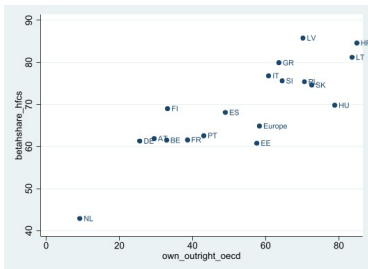
- Implications:

- $\beta_H^{j,N} \equiv \frac{P_{ht}^j \bar{H}_t^j}{Y_{Nt}} = \frac{P_{st}^j \bar{H}_t^j}{Y_{Nt}} \frac{1}{r_j + \delta_h - g}$.
- Note: $P_{st}^j \bar{H}_t^j = \gamma \mathcal{E}_t^j$. A change in \bar{H}_t^j has *no* effect on $\beta_H^{j,N}$.
- Now: Assume $\bar{H}_t^{od} \uparrow$ and $\bar{H}_t^{om} \downarrow$. Then $P_{st}^o \uparrow$ such that $P_{st}^o \bar{H}_t^{om}$ stays the same. But then $P_{st}^o \bar{H}_t^{od} \uparrow$

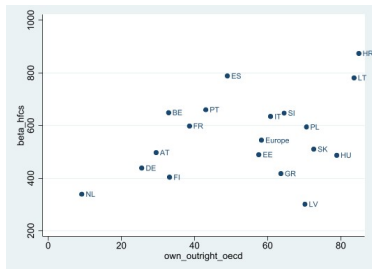
Outright owners

- **Wide variation** across countries: 15%-25% (AT, DE, NL) to > 75% (Eastern Europe).
- In the **UK** the share increased from 37% (1980) 41% (2018)
- Positive correlation of **outright ownership rates** with **aggregate wealth** and with **housing wealth share**.

◀ Add. implications



(a) β_H/β vs. outright owners



(b) β vs. outright owners

Capital gains savers (Fagereng et al., 2019)

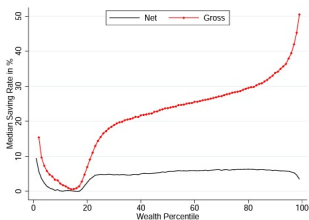
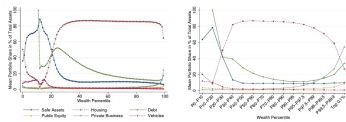


Figure 1: Saving rates across the wealth distribution.

(a) Net vs. gross savings



(a) Mean portfolio shares by wealth percentiles

(b) Mean portfolio shares by wealth percentiles

Note: The figure displays the mean portfolio share in percent of total assets across the wealth distribution, by percentile group. Safe assets is the sum of deposits, bonds, and informal loans. Debt is the sum of private debt and debt held indirectly via private firms. Public equity is the sum of directly-held stocks, stock funds, and stocks held indirectly via private firms. Private business is the book value of private firms, taking out public stocks and debt.

(b) Portfolio shares

◀ Add. implications

Inheritance flow and aggregate savings ratio

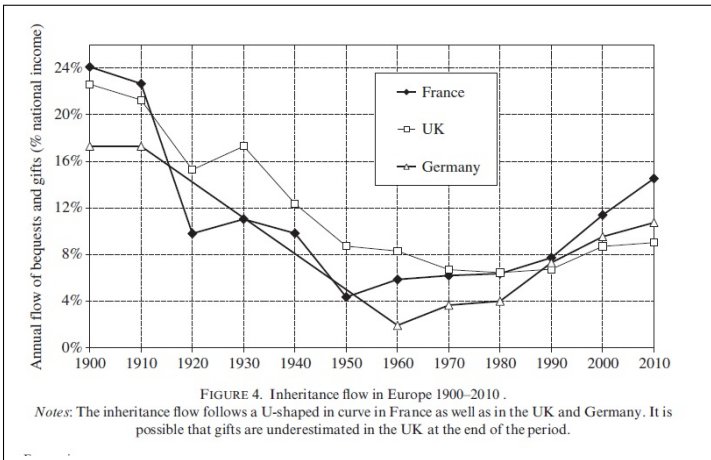
- **Bequest:**

- The **inheritance flow** is defined as $b_{yt}^N = \frac{B_t}{Y_{Nt}}$.
- The ratio of **financial bequests** moves only weakly from 5.9% to 7.1% in the 4 groups model. But this excludes the bequest of the directly owned housing stock.
- The mortality rate is given by $m = \frac{n}{e^{n(D-A)} - 1}$ which is about 1.36% in both situations. The **inherited directly owned housing stock** amounts to: $m \times \beta_{Hod}^N$. This adds 0.7pp to b_{yt}^N (initial) and 2.1pp (today)
- For data on some countries: [◀ See](#)

- **Savings:**

- The **aggregate gross savings rates** (including all capital gains) is $\bar{s} = 29.9\%$ (initial) which increases to $\bar{s} = 33.9\%$ (today).
- If one **excludes capital gains** from savings and from GDP: $\bar{s} = 28.2\% \rightarrow 31.4\%$
- The **net savings rate**: $\bar{s}^{net} = 15.3\% \rightarrow 15.1\%$ (with capital gains); $\bar{s}^{net} = 12.9\% \rightarrow 11\%$ (w/o capital gains)

Inheritance flows in Europa, 1900-2010



Source: Alvaredo et al., 2017