FAQ: How do I estimate the output gap?

Fabio Canova, Norwegian Business School, Norges Bank, and CEPR

March 2023

SUERF

Introduction

Great interest in potential (star) and gap variables in policy and academic arena. Definitions?

- Okun (1962): Potential is the maximum level of production with full employment that does not trigger inflationary pressures above ''the social desire for price stability and free markets" (point of balance between more output and greater price stability).

- Kuttner (1994): Potential is the production level generating constant inflation. Different from the maximum output level generated with any amount of aggregate demand.

- Friedman (1964) ''plucking model'': the business cycle is a cyclical contraction (due to negative demand shocks) from the maximum feasible output (driven by supply considerations).

- Woodford (2003): potential is the **counterfactual** path of the variables in a NK model when nominal frictions and monetary shocks are absent.

- **Gaps are meaningful objects only in terms of a model**.

• Many models availables (with real frictions, with financial frictions, with capacity constraints etc.). Time path of gaps depend on the model used (see e.g. Furlanetto et al., 2020).

• Misspecification important: many shortcuts used; many features left out.

• Could use a model robust approach, see Canova and Matthes (2021).

• Common to use statistical tools. Thought to be more robust to model misspecification; easier to produce estimates on a regular basis.

Questions

- How do DSGE-based gaps look like relative to, say, HP cycle or Blanchard and Quah transitory estimates?
- Is there a statistical approach uniformely minimizing the distortions?
- Is there a way to do better than existing statistical approaches?
- Can statistical estimates to provide guidance about policy questions?

Main points

- Theory potential need not be trending (in the sense of a unit root); may feature permanent and transitory fluctuations; and display important power at business cycle frequencies.

- Gaps may have important low frequency variations.

- Gaps and potentials correlated (driven by the same shocks) and display similar distribution of the variance by frequency.

- Statistical approaches inconsistent with the logic of NK-DSGE models.

- Butterworth filters potentially useful.

Illustration: Gali (2015)

$$\pi_t = \beta E_t \pi_{t+1} + k\tilde{y}_t \quad (1)$$

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t^n) + E_t\tilde{y}_{t+1} \quad (2)$$

$$r_t^n = \rho + \sigma\psi_y^n E_t(\Delta a_{t+1}) \quad (3)$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \quad (4)$$

v_t : AR(1) MP shock; a_t : AR(1) technology shock; $\tilde{y}_t = y_t - y_t^n$; $\beta =$ discount factor, $\rho = -\log(\beta)$, $\sigma =$ CRRA, $\psi_y^n = \frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha}$;$\psi =$ inverse Frish elasticity, $(1-\alpha)$ labor exponent in production function.

Solution for $(\tilde{y}_t, y_t^n)$

$$\tilde{y}_t = -(1-\beta\rho_a)\sigma\psi_y^n(1-\rho_a)\Lambda_a a_t - (1-\beta\rho_v)\Lambda_v v_t \quad (5)$$

$$y_t^n = \psi_y^n a_t + \gamma_y^n \quad (6)$$

where

$$\Lambda_v = \frac{1}{(1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y]+k(\phi_\pi-\rho_v)} > 0 \quad (7)$$

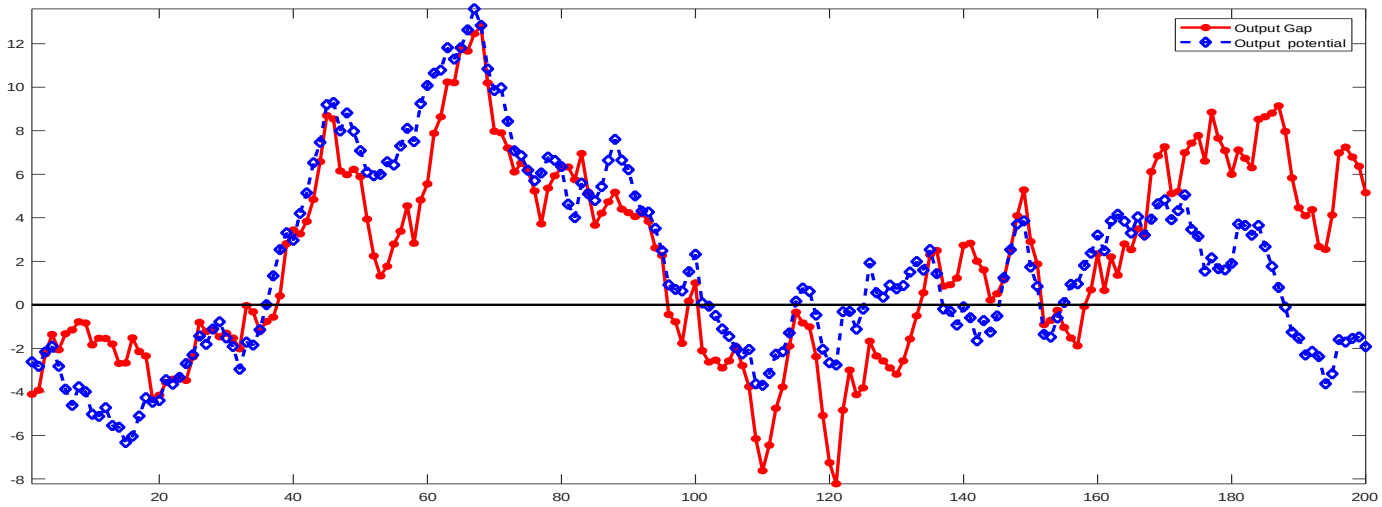
$$\Lambda_a = \frac{1}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+k(\phi_\pi-\rho_a)} > 0 \quad (8)$$

$$\gamma_y^n = \frac{-(1-\alpha)(\mu - log(1-\alpha))}{\sigma(1-\alpha)+\psi+\alpha} > 0 \quad (9)$$

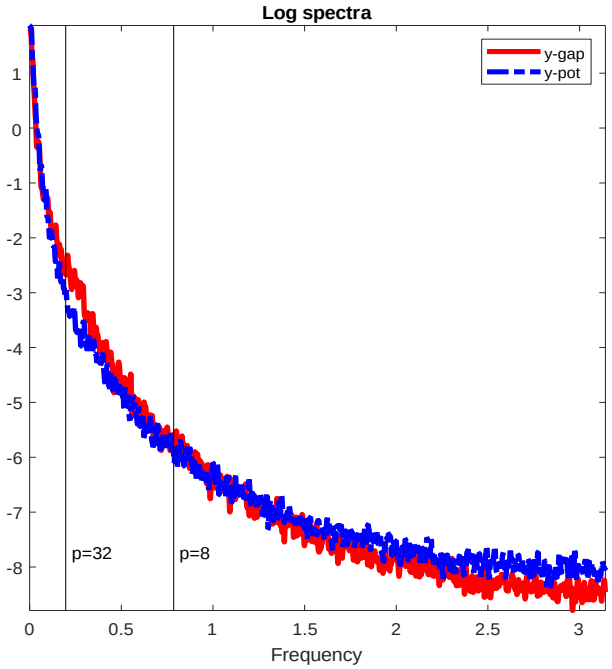
Takeaways

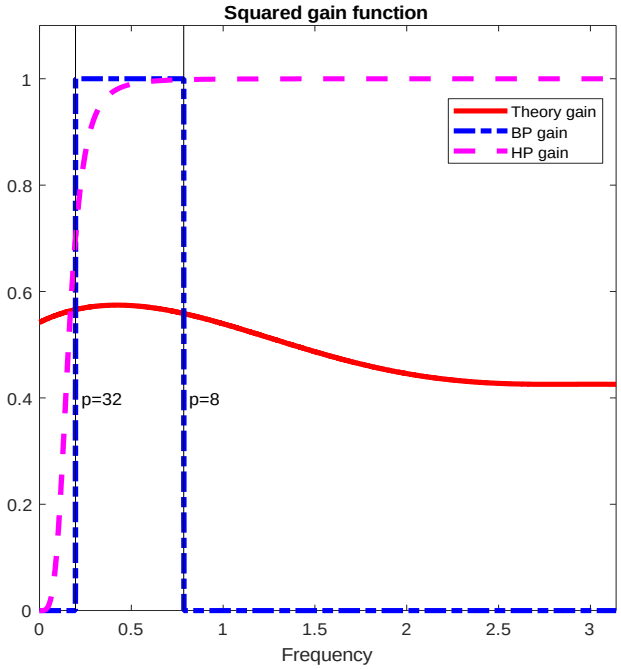
- Potential and gap correlated: a_t enters the $\tilde{y}_t$ and y_t^n
- If a_t is persistent, both gap and potential will display similar persistence.
- If ρ_a greater than ρ_v, gap and potential will have similar spectral shape.
- Special case $\rho_a = 1$: gap influenced only by v_t.
- Generic problem: solution for $(\tilde{i}_t, r_t^n)$ has similar features.
- Same conclusion with more shocks (investment specific, goverment spending, labor supply, markup, etc). Persistence of components driven by the process with largest AR root.

SW model



corr(gap, potential)$=$ 0.78; $\rho_{gap} = 0.96$; $\rho_{potential} = 0.97$.


Log spectra
y-gap
y-pot
p=32
p=8
Frequency


Squared gain function
Theory gain
BP gain
HP gain
p=32
p=8
Frequency

• Statistical methods assume that latent components:

1) are generally uncorrelated (exception BN, some UC);

2a) persistence of potential larger than the persistence of the gap.

2b) gap and potential located at different frequencies of the spectrum.

• **Lack of theory-practice consistency produces large distortions**.

Horse race 1: Gap extraction

- **Polynomial filtering** the *least distorting* filter because
- Frequency distribution of the variance of the gaps undistorted.
- Estimated gap displays low frequency variations.
- Relative ranking independent of the sample size and filters' parameters.

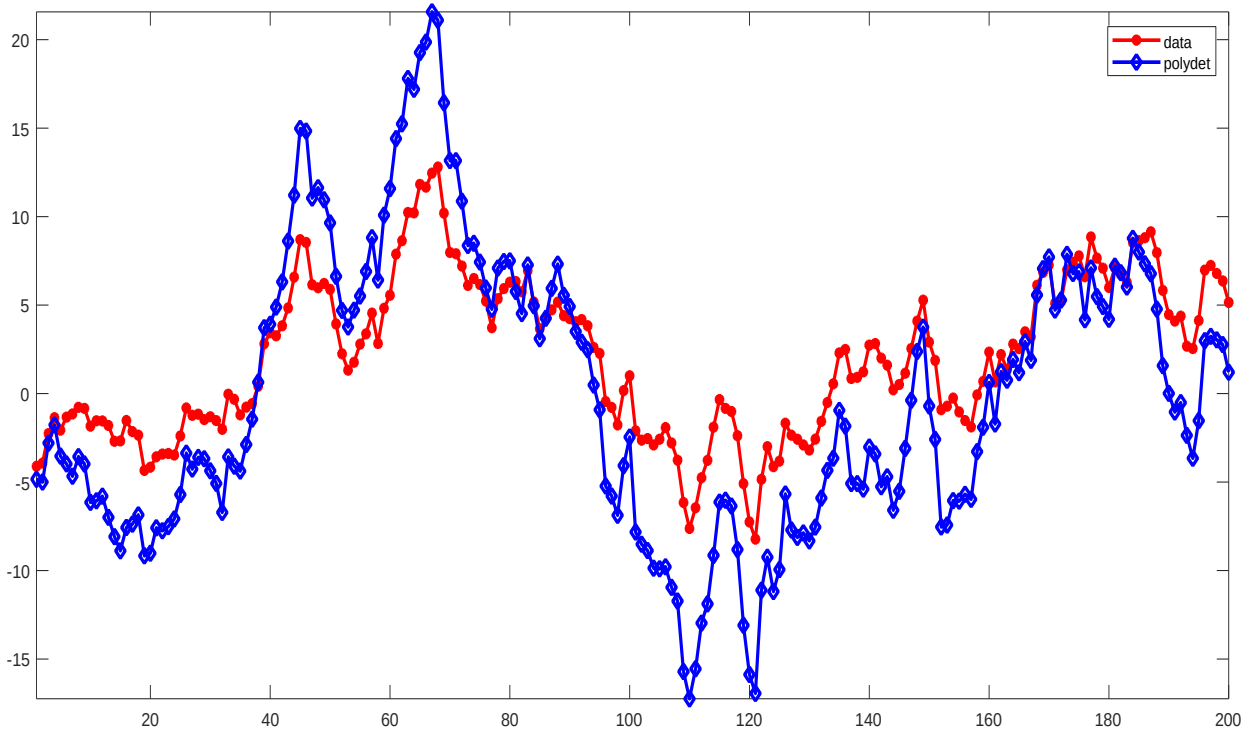
Horse race 2: Transitory fluctuations extraction

• **Differencing** is the *least distorting* filter; **Polynomial filtering** close second.

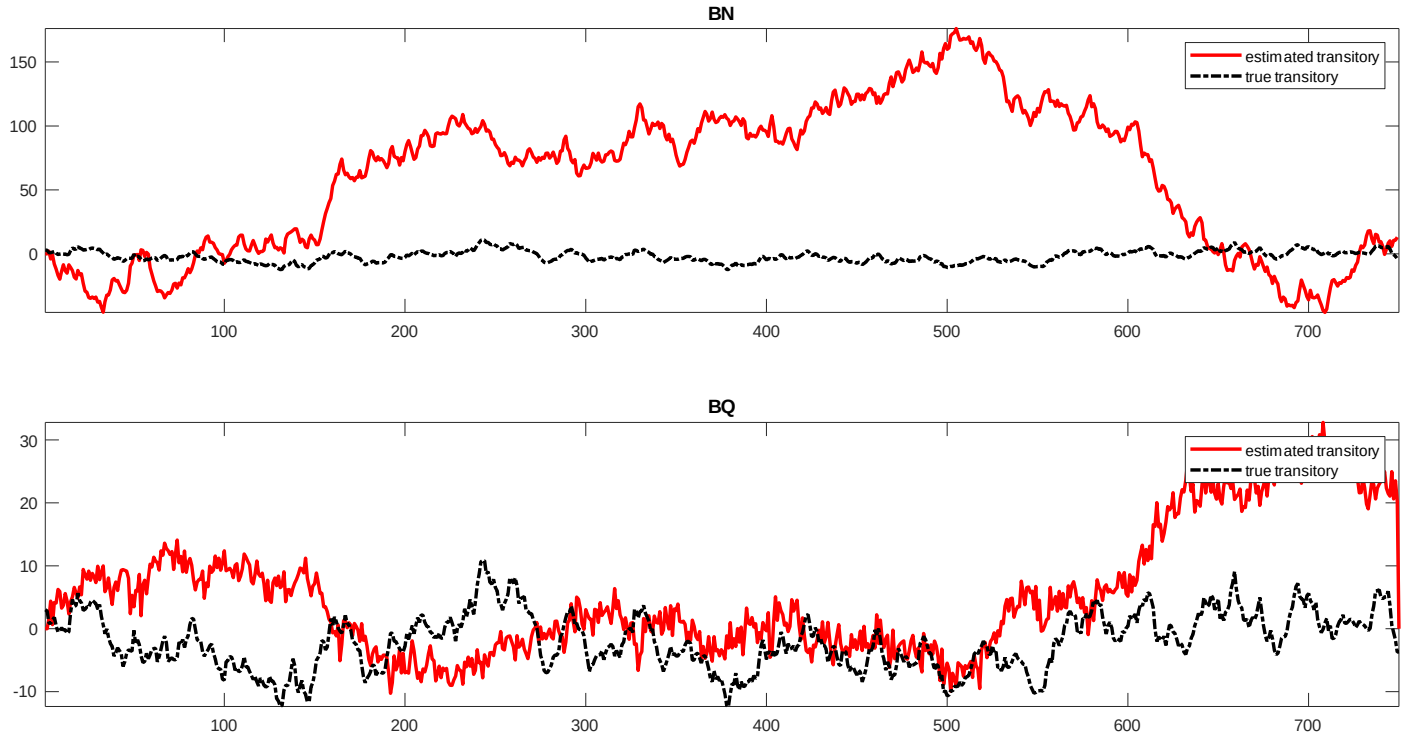
• Distortions typically larger because at business cycle and high frequencies permanent fluctuations matter.

• Small samples affect the ranking; the parameters of the filters do not.

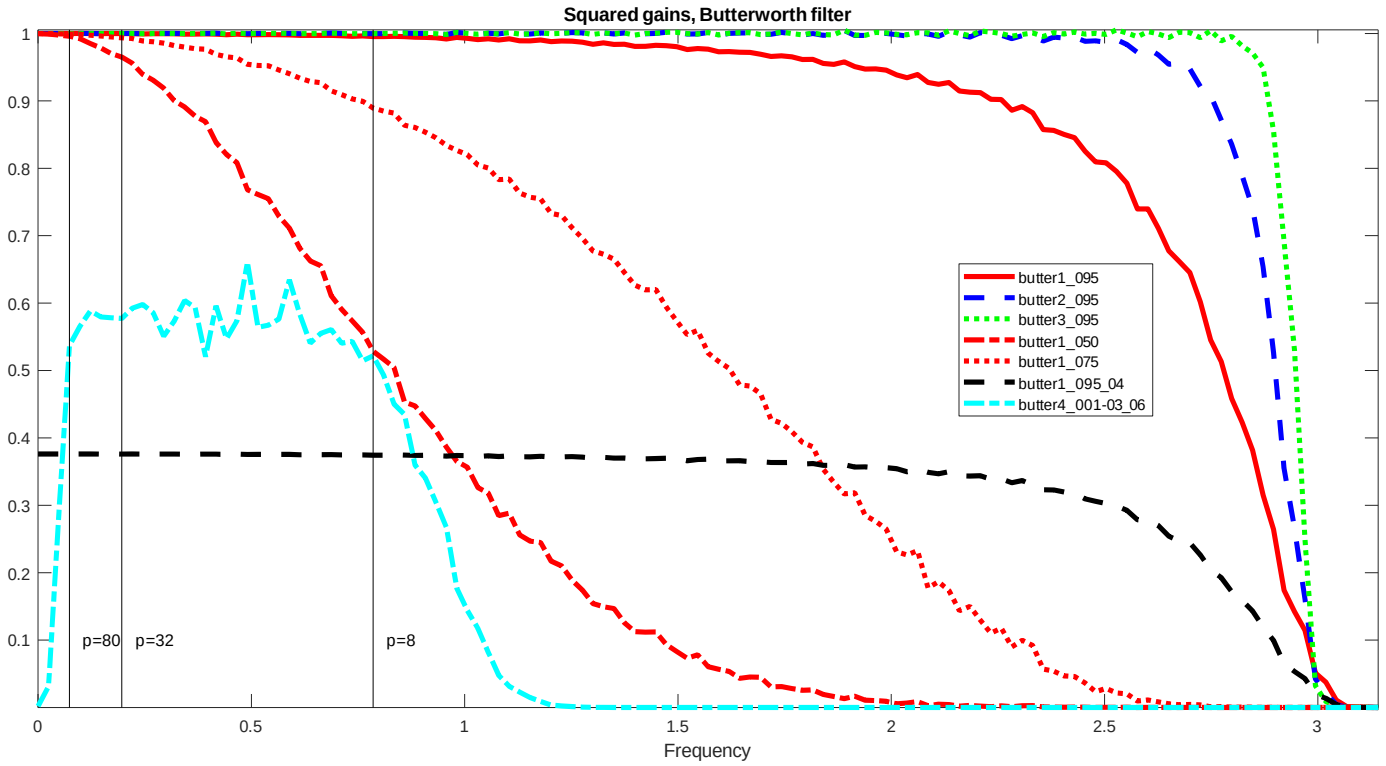
Polynomial estimate and true gap



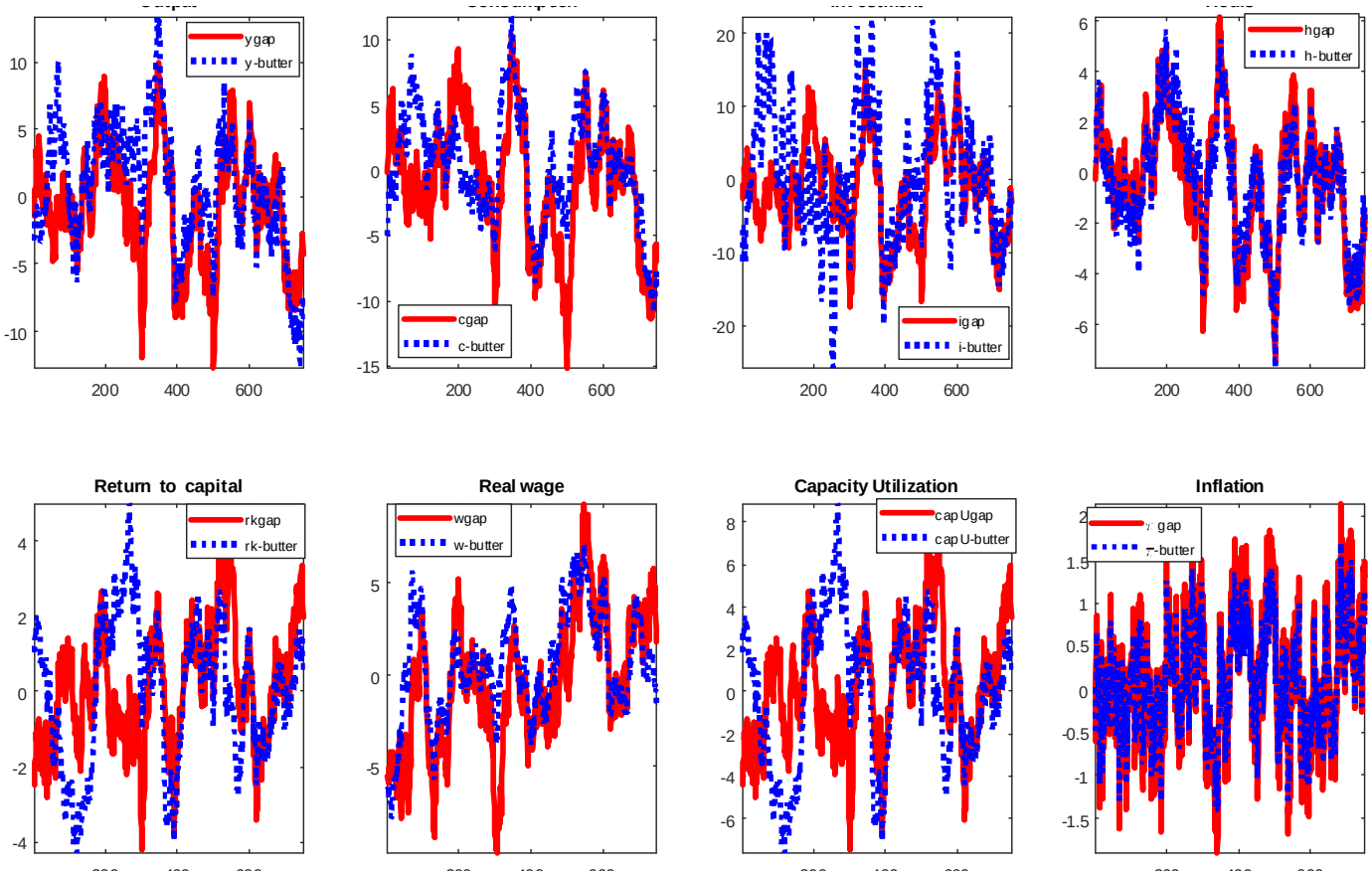
BN-BQ estimates and true gap



• Are there filters consistent with the properties of gaps and potentials in NK DSGEs? A Butterworth (BW) class.



- In simulation exercises BW preferable to others to extract gaps.



• With real data if the labor share is used to judge the reasonableness of the decomposition, BW filter is good.

Table 1:Matching the US labor share, summary results

Statistic	RMSE(all)	RMSE(low)	RMSE(bc)	Corr(all)	Corr(low)	Corr(bc)	Persistence	Variability
POLY	0.1500	0.0217	0.0157	0.4423***	0.1900	0.5845**	0.0019	0.0300
HP	0.0561	0.0072	0.0127	0.4430***	0.2712	0.5824	-0.0007	-0.0052
FOD	0.0560	0.0064	0.0076	0.4364	0.1991	0.5796	-0.0002	-0.0088
LD	7.9553	0.2814	0.0427	0.1886	0.1558	0.2750	0.2639	1.5606
BP	0.0554	0.0064*	0.0139	0.2398	0.0988	0.5659	-0.0007	-0.0061
Wa	2.5607	0.0900	0.0220	0.1349	0.1521	0.3344	0.0843	0.4918
Ham	0.1588	0.0236	0.0377	0.3746	0.2614	0.5307	0.0000*	0.0266
UC	0.1002	0.0093	0.0074**	0.3530	0.1521	0.5621	0.0017	0.0054
BN	56.0487	2.2333	0.4567	0.2214	0.4534*	0.3260	1.7990	11.0246
BQ	2.4840	0.1221	0.0459	0.3097	0.3506	0.4810	0.0811	0.5715
BW	0.0631*	0.0095	0.0074**	0.4405***	0.1900	0.5845**	0.0000*	-0.0023*