Robust Real-Time Estimates of the German Output Gap based on a Multivariate Trend-Cycle Decomposition

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The views expressed here represent the authors' personal opinions and do not necessarily reflect the views of the German Council of Economic Experts.

In a nutshell

- Output gap is important measure of slack in an economy
- Usually computed from GDP at quarterly frequency
- We present a model for nowcasting the German output gap at in the spirit of Berger, Morley and Wong (forthcoming)
- Our model is very reliable in real-time

Beveridge-Nelson Zerlegung

- BN Trend:

$$\tau_t = \lim_{h \to \infty} \mathbb{E}_t \left[y_{t+h} - h \cdot \mu \right].$$

- evaluation with vector autoregression:

$$\boldsymbol{Y}_t = \boldsymbol{F} \boldsymbol{X}_{t-1} + \boldsymbol{\epsilon_t}.$$

- According to Morley (2002) and Morley and Wong (2020), for the BN cycle c_t to the variable at location k, we obtain

$$c_t = -\mathrm{s_k} \boldsymbol{F} (\boldsymbol{I} - \boldsymbol{F})^{-1} \boldsymbol{X}_{\mathrm{t}}.$$

- $\mathrm{s_k}$ is a selector row vector, $\boldsymbol{F}$ is the coefficient companion matrix, and $\boldsymbol{X}_t$ contains variables centered around zero, and lags

MF-BVAR(p)

- We stack the k-th monthly variables in time $t - v$:

$$\boldsymbol{m}_{t-v} = \begin{bmatrix} \tilde{m}_{1,t-v} \\ \tilde{m}_{2,t-v} \\ \vdots \\ \tilde{m}_{k,t-v} \end{bmatrix}$$

- example: monthly variables $\boldsymbol{m}$ within a quarter and real GDP growth $\Delta\tilde{y}_t$ in 2020Q1:

$$\boldsymbol{Y}_{2020\text{Q1}} = \begin{bmatrix} \boldsymbol{m}_{\text{Jan2020}} \\ \boldsymbol{m}_{\text{Feb2020}} \\ \boldsymbol{m}_{\text{März2020}} \\ \Delta\tilde{y}_{2020Q1} \end{bmatrix}$$

Data

- We sample 8 quarterly and 23 monthly indicators to approximate labour and capital inputs:
  - capital-related aggregates
    - real economy
    - financial market
    - external relations
    - fiscal policy
  - labour market aggregates
- Bundesbank data; covering 1995M1 - 2022M6.
- We consider $p = 4$ quarterly lags in the MF-BVAR

Rule-based Model Selection

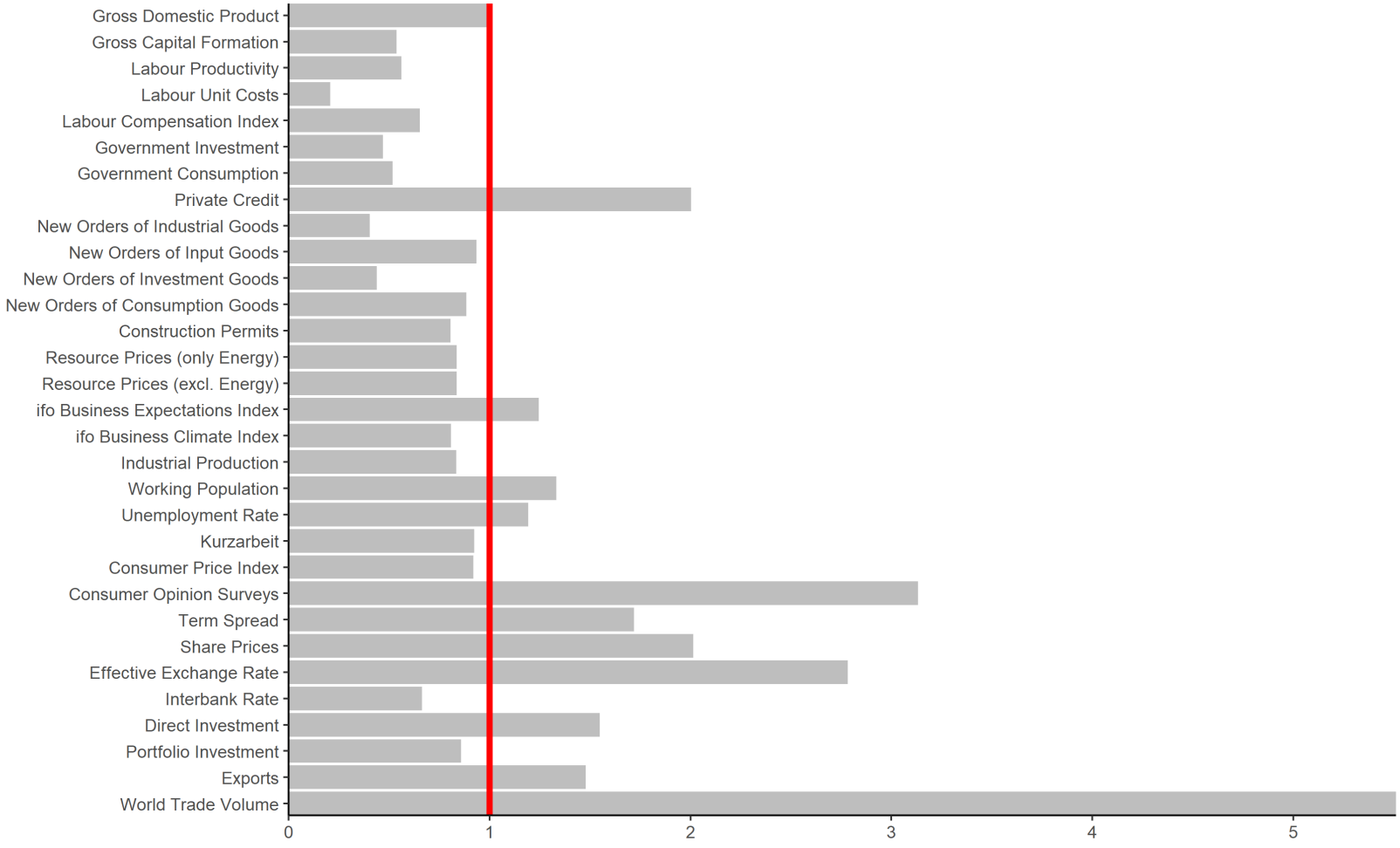

Figure 1: Normalized time-averages of standard deviations of informational decomposition. The red line indicates the standard deviation of the GDP growth contribution to output gap variation.

The German Output Gap

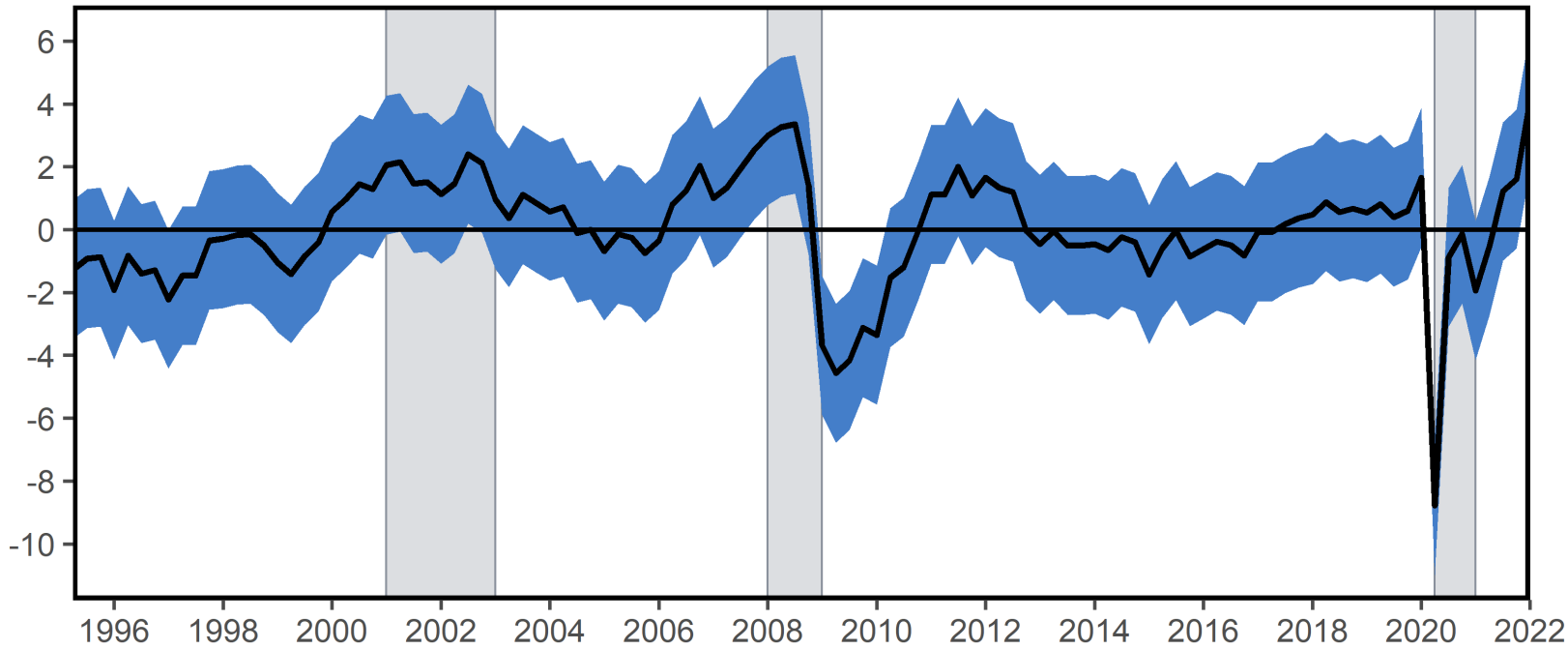

Figure 2: German output gap in percentage deviation from potential output, 1995Q1 - 2022Q1 (black). Blue and orange shaded areas indicate 90% credible sets following Kamber et al. (2018) for the mean estimate and the nowcast, respectively. Grey areas indicate recessions according to the German council of economic experts and the COVID-19 pandemic.

Comparison with Alternative Estimates

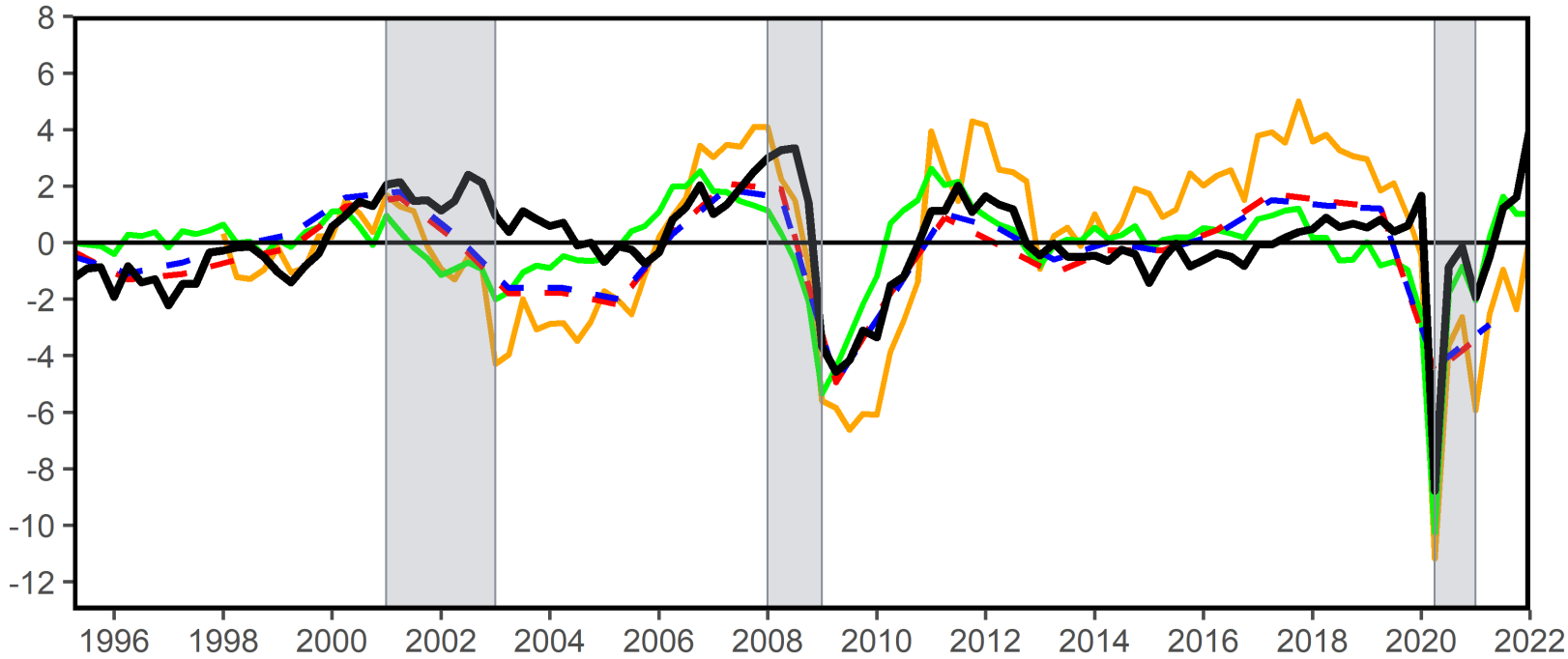

Figure 3: Comparison of different estimates of the German output gap:

- our estimate (black)
- one-sided HP filter (green)
- Hamilton filter (yellow)
- German Council of Economic Experts (red, dashed)
- AMECO (blue, dashed)

Informational Decomposition

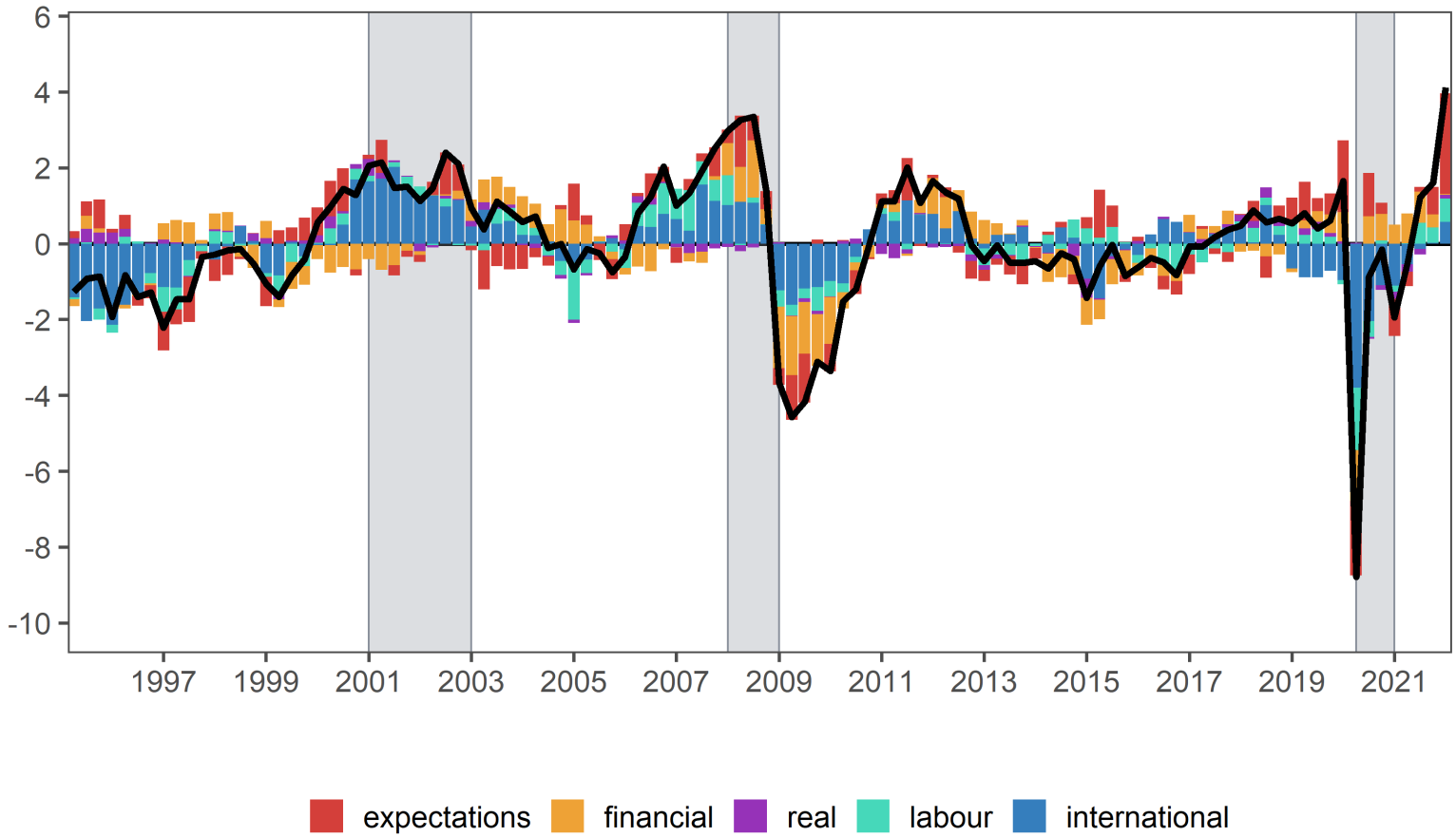

Figure 4: Informational decomposition of the German output gap. The financial block comprises private credit, share prices, the term spread, the real economy block summarizes contributions from gross domestic product, the labour block is made up of unemployment and the working population, the expectations block comprises consumer sentiment and business expectations and the international block contains the direct investment, the exchange rate, exports and world trade. For further notes see Figure 7.

Nowcast

COVID-19

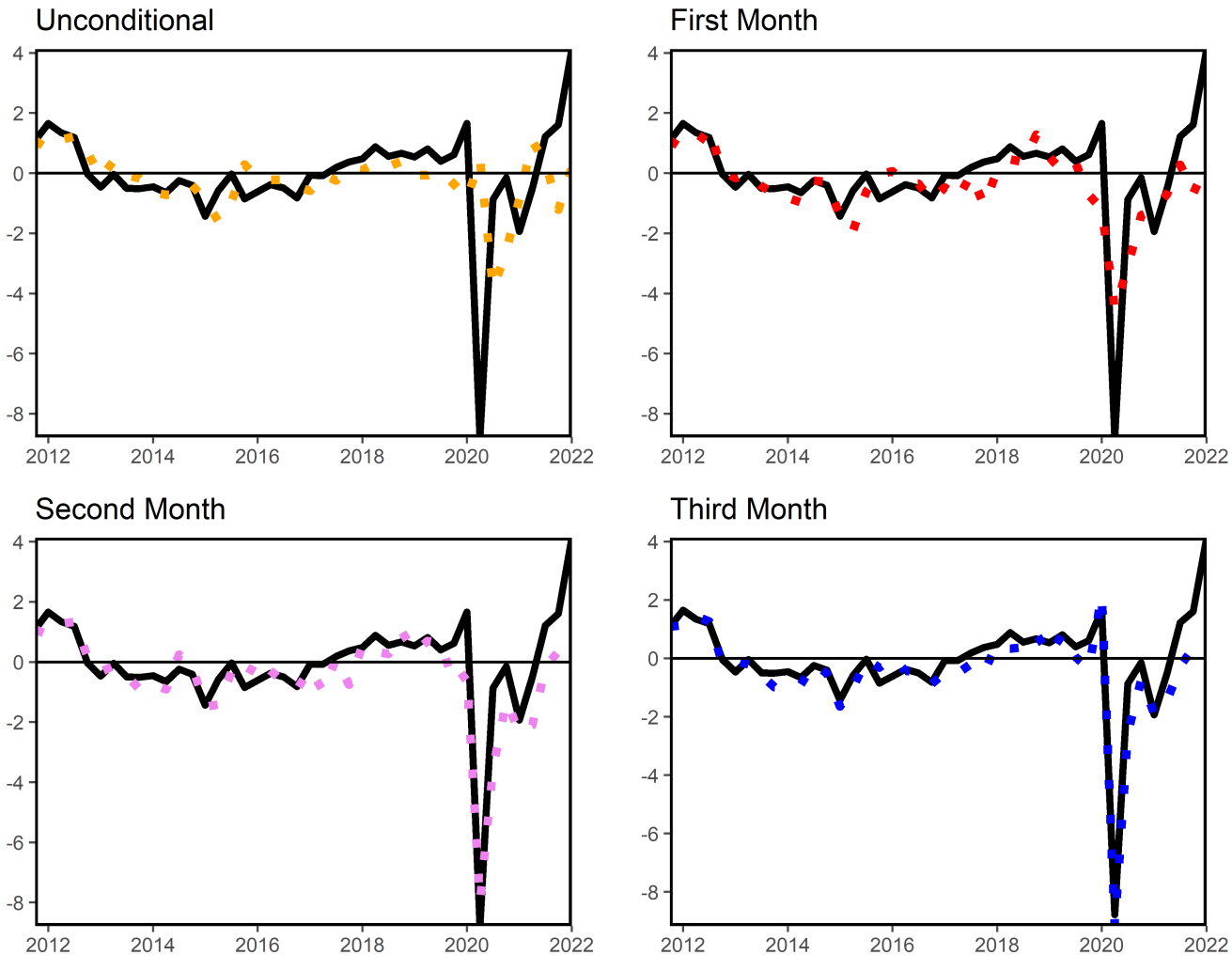

Figure 5: Nowcasts since 2012. Ex post estimate (black), unconditional forecast (orange, upper left panel) and nowcasts from the perspective of month 1 (red, upper right panel), month 2 (pink, lower left panel), month 3 (blue, lower right panel). Vintage GDP data is used, parameters are estimated and the specification is chosen in real-time (i.e. every quarter).

Real-time reliability

	Mean	Sd	Min	Max	RMSD	AR(1)
y_t - y_t^1	-0.01	0.05	0.04	-0.07	0.07	-0.01
y_t - y_t^2	0.01	0.04	0.04	-0.13	0.07	0.01
y_t - y_t^3	0.58	0.64	0.61	-0.33	1.53	0.52

Table 1: Summary and loss statistics in the spirit of Orphanides and van Norden (2002) of the difference of the baseline output gap y_t and the result taking into account parameter revisions (y_t^1), GDP data-revisions (y_t^2) as well as real-time model selection (y_t^3). From left to right: mean, standard deviation, minimum, maximum, root mean squared difference and the AR(1) regression coefficient of y_t - $y_t^{\{1,2,3\}}$, none of which is significantly different from zero at conventional levels.

Conclusion

- We use a mixed-frequency BVAR to nowcast the German output gap before observing current real GDP data
- Monthly indicators contain useful information about the current output gap
- Expectations as well as international and labour market variables contribute most to (reduced-form) variation of the German output gap
- Our nowcasting scheme is precise and very robust

Thank you.

Appendix

Pseudo-Real-Time Experiment 1: Parameter Revisions

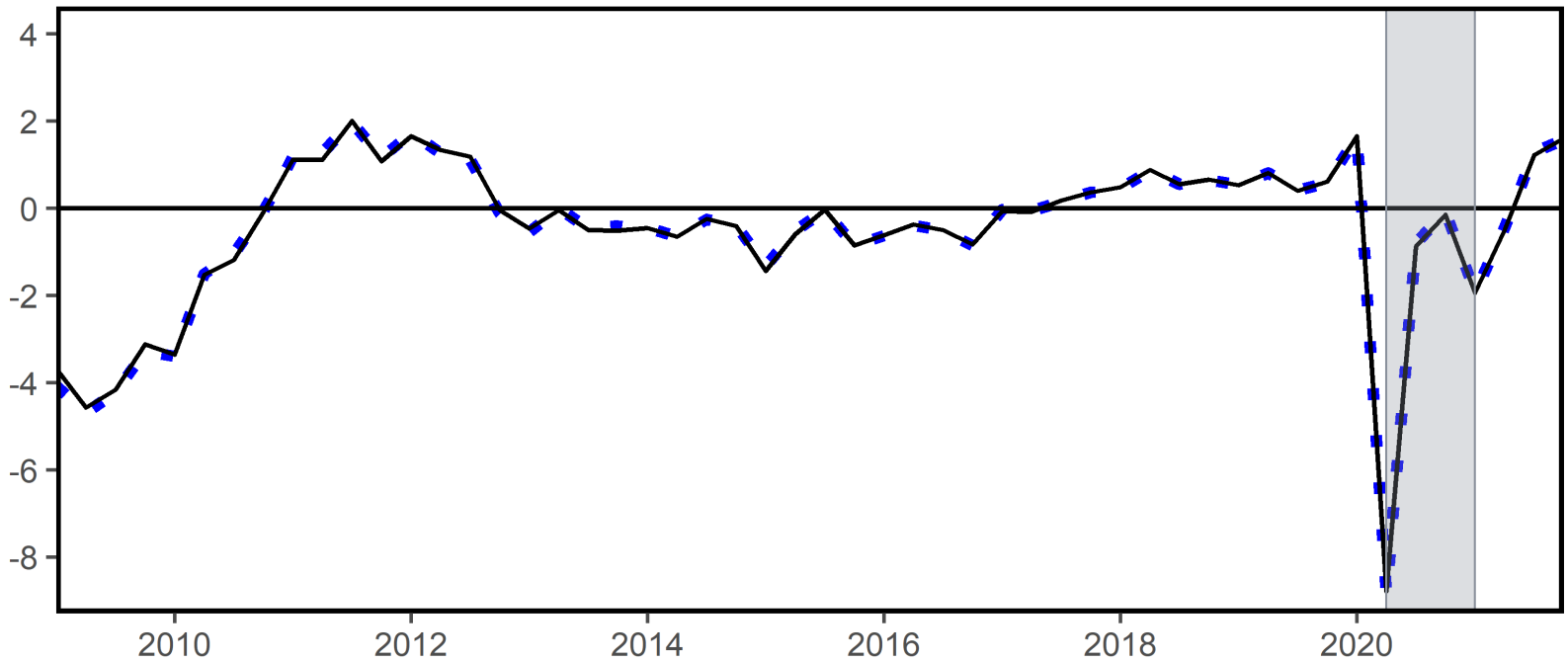

Figure 6: Black: Ex post estimate. Blue: Estimate with parameter revisions.

Appendix

Pseudo-Real-Time Experiment 2: GDP-Revisions

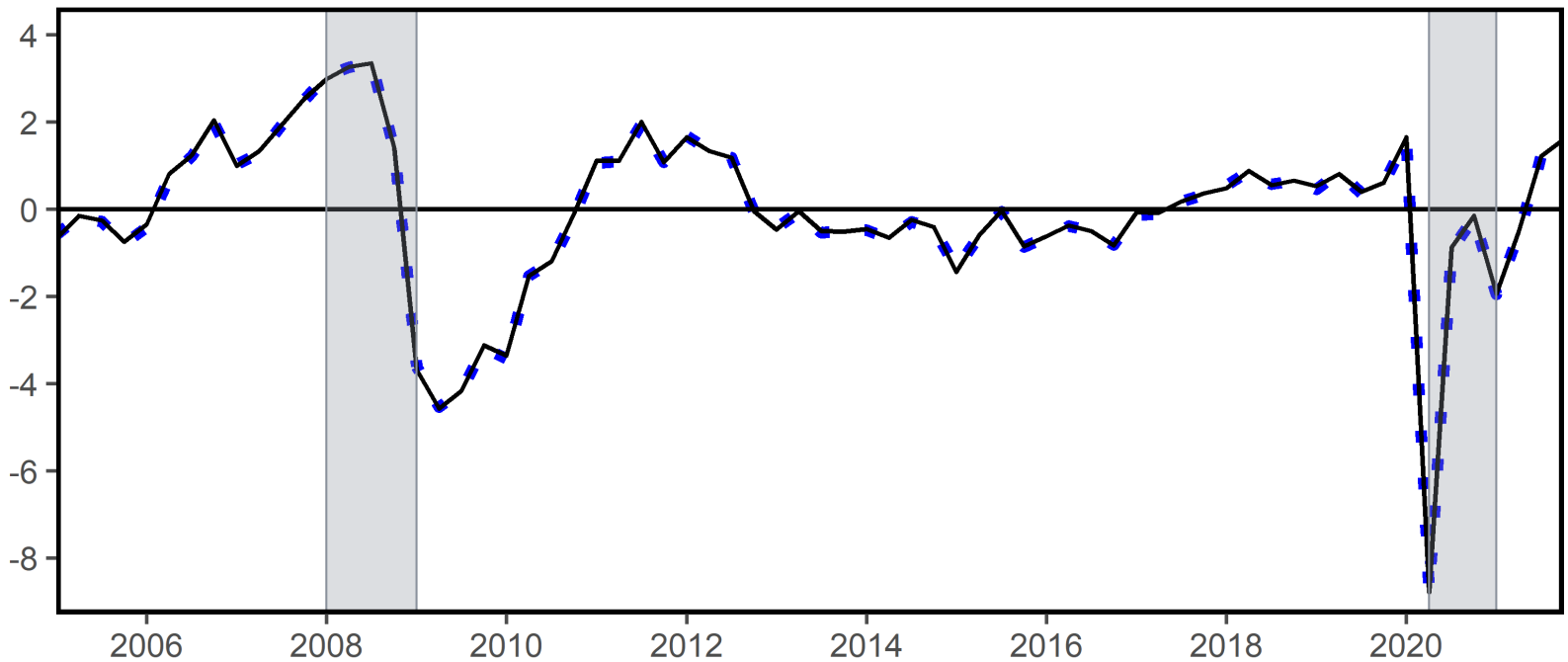

Figure 7: Black: Ex post estimate. Blue: Estimate with GDP revisions.

Appendix

Real-Time Experiment: Model Selection

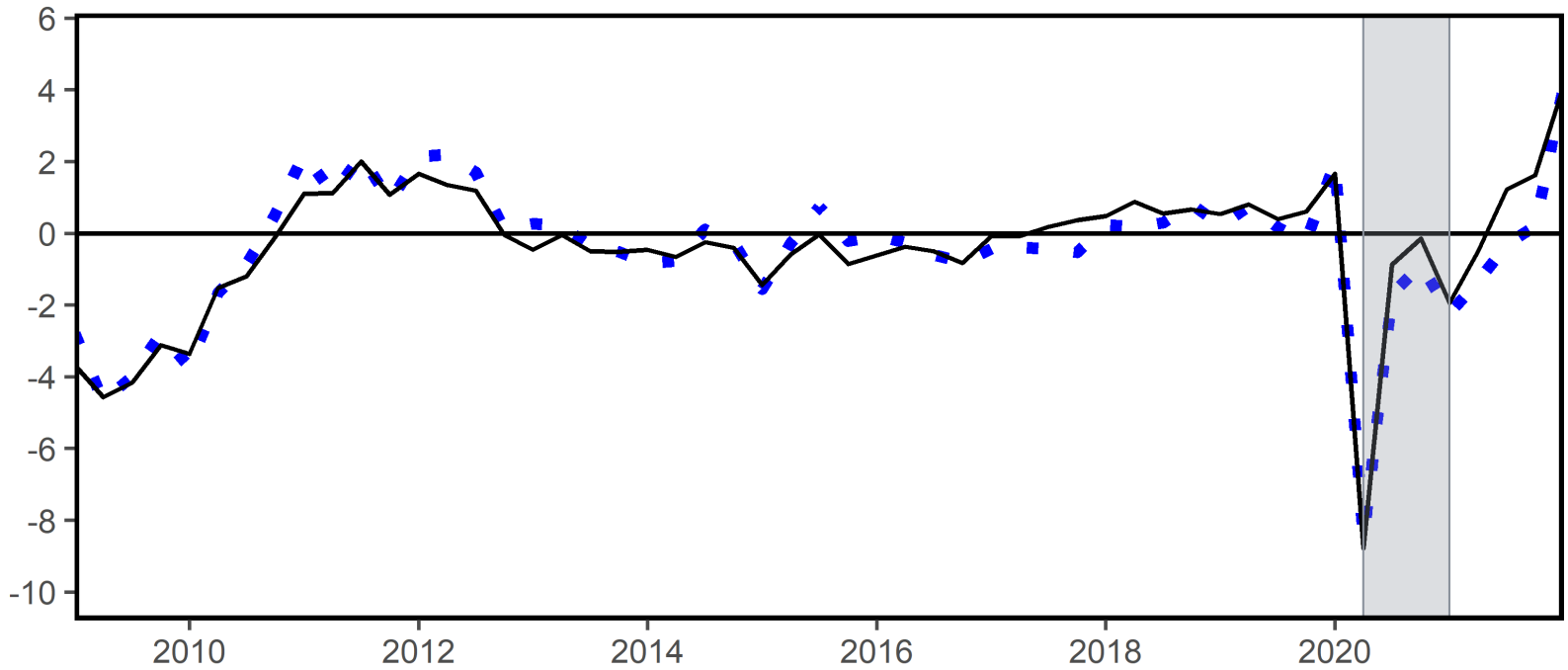

Figure 8: Black: Ex post estimate. Blue: Estimate with quarter-on-quarter model selection.

Appendix: Nowcasting

Performance

	1^{st} **month**	2^{nd} **month**	3^{rd} **month**
Share prices	0.39	0.28	0.12**
Term spread	0.38	0.26*	0.11**
Consumer opinion surveys	0.34**	0.16**	0.06**
Business expectations	0.28	0.15	0.04
Unemployment rate	0.31*	0.15**	0.05**
Working population	0.29*	0.15	0.05
World trade volume	0.43**	0.28	0.15
Exports	0.41	0.28	0.15
Direct investment	0.40	0.28	0.14
Effective exchange rate	0.40	0.27	0.14

Table 2: Within-quarter mean absolute forecast errors associated with monthly variables rounded to two decimal places for all four models, given the full-sample parameters. ** and * indicate Diebold-Mariano p-values (based on mean absolute error) equal to or smaller than 0.05 and 0.10, respectively. Variables are ordered by expected release.

Appendix: Nowcast & variance decomposition

The nowcast for the variable k is

$$c_{t+1} = -\boldsymbol{s}_k \boldsymbol{F}(\boldsymbol{I} - \boldsymbol{F})^{-1}[\boldsymbol{F}\boldsymbol{X}_t + \boldsymbol{H}\boldsymbol{\varepsilon}_{t+1}]$$

- We assume for $c_{t+1t+\omega}$ (for $\omega \in (0,1)$) that $\varepsilon_{t+1|t+\omega} \neq 0$
- We decompose the variation of the output gap with the help of

$$c_{ij,t} = -\sum_{l=0}^{t-1} \boldsymbol{s}_k \boldsymbol{F}^{l+1}(\boldsymbol{I} - \boldsymbol{F})^{-1} \boldsymbol{H} \boldsymbol{s}_j' \boldsymbol{s}_j \varepsilon_{t-1}$$

Appendix: MF-BVAR(p)

$$\boldsymbol{Y}_t = \Phi_1 \boldsymbol{Y}_{t-1} + \Phi_2 \boldsymbol{Y}_{t-2} + \ldots + \Phi_p \boldsymbol{Y}_{t-p} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma)$$

- Minnesota Prior as in Morley and Wong (2020) for lag l:

$$\begin{aligned} \mathbb{E}[\phi_l^{j,k}] &= 0 \\ Var[\phi_l^{j,k}] &= \begin{cases} \frac{\lambda^2}{l^2}, & j = k \\ \frac{\lambda^2}{l^2} \frac{\sigma_j^2}{\sigma_k^2}, & \text{sonst} \end{cases} \end{aligned}$$

- We set λ by minimizing the forecast RMSE for GDP growth.

Appendix

Data – More detail

Variable	Transformation	Frequency
CAPITAL		
External Relations		
CPB World Trade Monitor: World Trade Volume	rolling demean	M
Current Account: Exports	growth rates	M
Capital account Balance: Portfolio Investment	growth rates	M
Capital Account Balance: Direct Investment	growth rates	M
Real Effective Exchange Rate of the Euro against EERK-42	rolling demean	M
Finance		
Interbank Rate for Germany (obtained from FRED database)	growth rates	M
Total Share Prices for All Shares for Germany (obtained from FRED)	rolling demean	M
Term spread (1 year over 10 year government bonds)	growth rates	M
Non-financial private sector credit (obtained from BIS)	growth rates	Q
Fiscal Activity		
Government Consumption	growth rates	Q
Government Investment	growth rates	Q

Table 3: Variable blocks and data transformations. 'Growth rates' denotes the transformation 100 $\times$ first differences of natural logarithms. 'rolling demean' denotes a rolling demean (backward moving average) filter with a 40-quarters window. 'M' and 'Q' denote monthly and quarterly frequency, respectively.

Appendix

Data – More detail

Variable	Transformation	Frequency
CAPITAL		
Sentiment and Expectations		
OECD Consumer Opinion Surveys (obtained from FRED)	growth rates	M
ifo Business Climate Index (obtained from ifo Institute)	growth rates	M
ifo Business Expectations Index (obtained from ifo Institute)	growth rates	M
Real Economy		
Consumer Price Index	growth rates	M
Construction Permits	growth rates	Q
Industrial Production	growth rates	M
New Orders of Consumption Goods	growth rates	Q
New Orders of Investment Goods	growth rates	Q
New Orders of Input Goods	growth rates	Q
New Orders of Industrial Goods	growth rates	Q
Real Gross Domestic Product	growth rates	Q
Resource Price Index (excl. Energy)	growth rates	Q
Resource Price Index (only Energy)	growth rates	Q
LABOUR		
Hours in Construction	growth rates	M
Labour Compensation Index	growth rates	Q
Labour Unit Costs	growth rates	Q
Labour Market Stabilization Policy ('Kurzarbeit' policy)	growth rates	M
Unemployment Rate	growth rates	M
Working Population	growth rates	M

Table 4: Data cont'd

Appendix

Robustness – Alternative Information Set

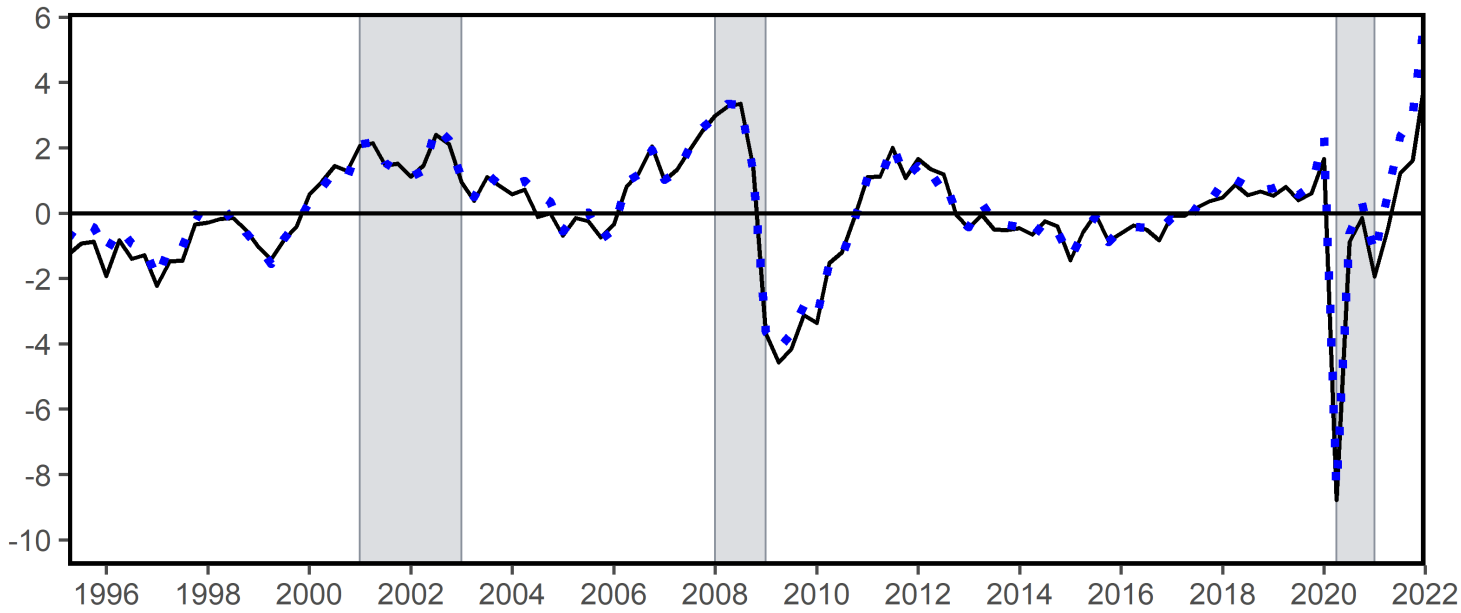

Figure 9: Alternative output gap estimate (blue) and baseline estimate (black). In addition to the baseline variables, we include consumer price index, Kurzarbeit and indices for new order for consumption goods and input goods.

Appendix

Choleski Decomposition

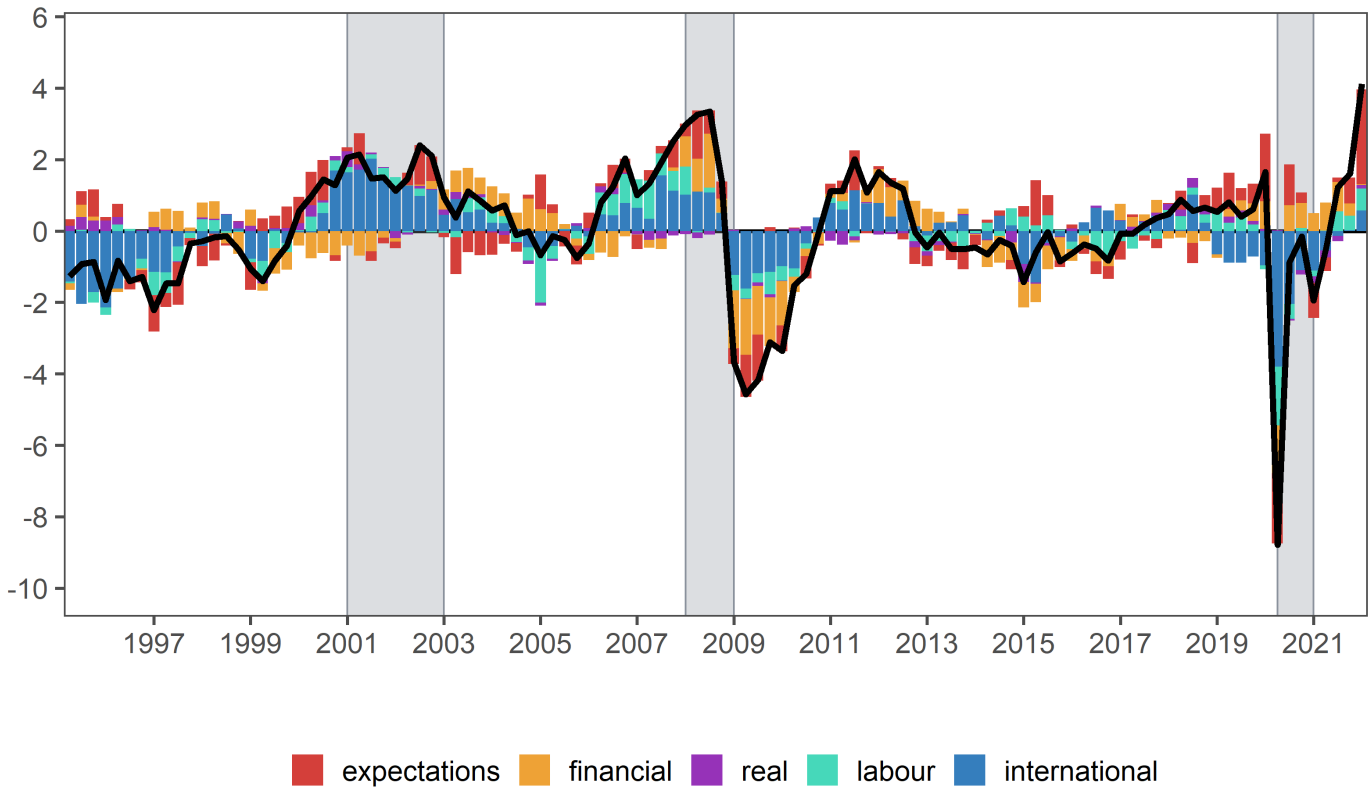

Figure 10: Structural historical decomposition based on Choleski decomposition of the German output gap. For further notes see Figure 4.