Undesired Consequences of Calvo Pricing in a Non-linear World

Aleš Maršál, Katrin Rabitsch, Lorant Kaszab
## (NBS, WU Vienna, MNB)

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Contents

Need for non-linear modeling approach

- traditionally, macroeconomic dynamics arising from the Calvo framework have been studied primarily in terms of (log-)linear approximations to the true model solution
- Recent developments in many fields of macroeconomics increasingly require non-linear solution techniques

Large shocks

The burst of high inflation
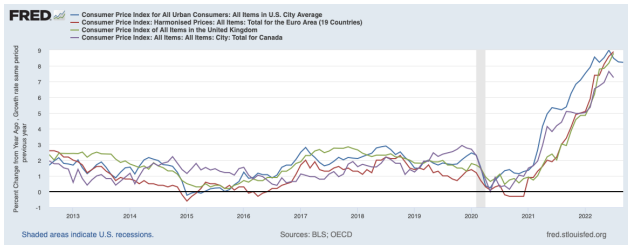
Makroekonomické ukazatele
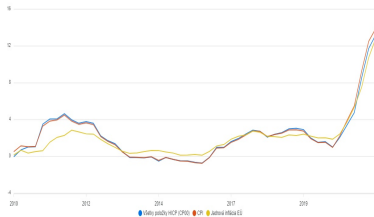
Inflation widens distribution of prices

Candia et.al. (2021), US firms although distinct from HH feature similar pattern. Klaus Adam inefficient price dispersion

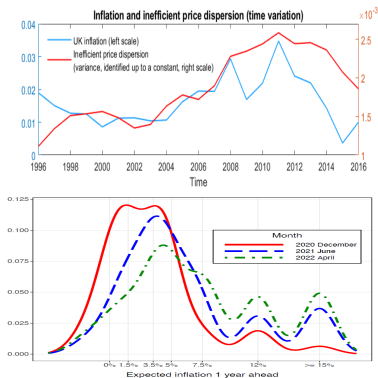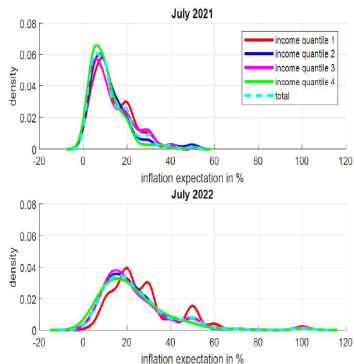Figure: Dispersion inflation and expectations

Figure: SK Household Survey Inflation Expectations

Inflation uncertainty

... impacts consumption saving decision

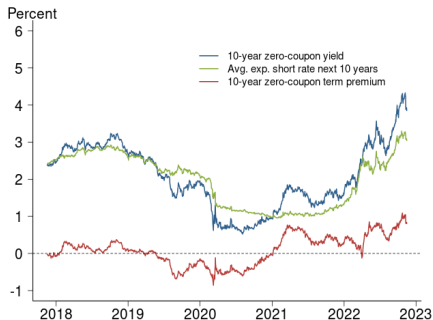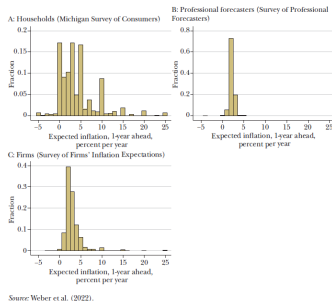Figure: Treasury yield premium model

Figure: Cross-sectional Dispersion in Expectations

Mankiw, Reis, and Wolfers (2004), Weber et.al. (2022) report significant disagreement in expectations of HH's, firms and professional forecasters. Drenik and Perez (2014) show uncertainty increases price dispersion

Trend inflation, $\pi = \pi^{trend} + \pi^c$

Figure: 10Y inflation expectations in US (markets)
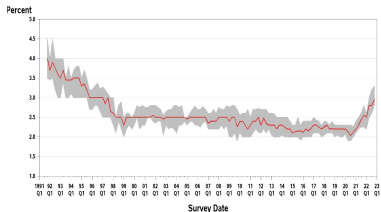
Figure: 10Y inflation expectations from Swaps
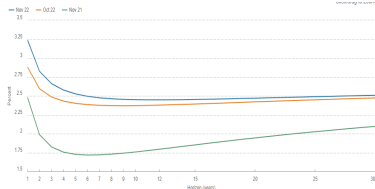
Source: Federal Reserve Bank of Cleveland calculations based on data from Blue Chip, Bloomberg, Bureau of Labor Statistics, Federal Reserve Bank of Philadelphia, Federal Reserve Board, Haver Analytics, and the model of Haubrich, Pennacchi, and Ritchken, 2012 "Inflation Expectations, Real Rates, and Risk Premia: Evidence from Inflation Swaps," Review of Financial Studies, 25(5).

Figure: Inflation Expectations in EU
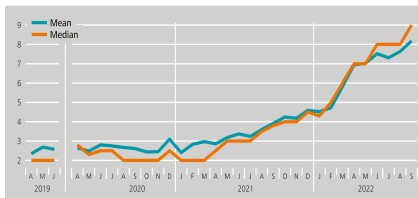
Source: Deutsche Bundesbank Online Panel Households (BOP-HH). * Question: What do you think the rate of inflation/deflation

Figure: Inflation Expectations Slovakia
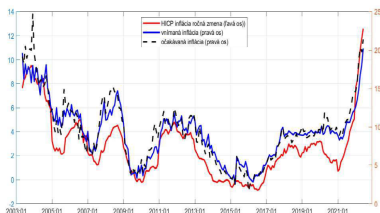
Where are we standing . . . ?

- ▶ Inflation is far from long-run mean
- ▶ Prices are more dispersed across products
- ▶ Higher uncertainty about future inflation
- ▶ Trend inflation

Modeling framework

- ▶ What are the implications for monetary policy in this *new* environment?
- ▶ To answer this question in standard monetary macro framework: need to use non-linear model setting!
- ▶ NK DSGE model with Calvo pricing and trend inflation solved non-linearly
- ▶ Calvo pricing is the dominant method to introduce nominal rigidity

Prescription for monetary policy

...to anchor inflation expectations

Taylor Principle, $\phi_\pi > 1$

$$i_t = \phi_\pi \pi_t \tag{1}$$

Generalized Taylor principle (Determinacy region)

$$i_t = \bar{\imath} + \phi_\pi(\pi_t - \pi^{trend}) + \phi_y(y_t - y_t^*) \tag{2}$$

Stability region, $S_t < S_{t-1}$

- ▶ non-linear counterpart to the determinacy region
- ▶ need for MP to contain dispersion of prices in the economy

$$S_t^{\frac{1}{1-\theta}} = (1 - \zeta) \left[\frac{1 - \zeta(\Pi_t)^{\epsilon-1}}{1 - \zeta} \right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}} \tag{3}$$

Monetary Policy

Anchoring inflation expectations and preventing price-inflation spiral

$$i_t = \bar{i} + \phi_\pi(\pi_t - \pi^{trend}) + \phi_y(y_t - y_t^*)$$
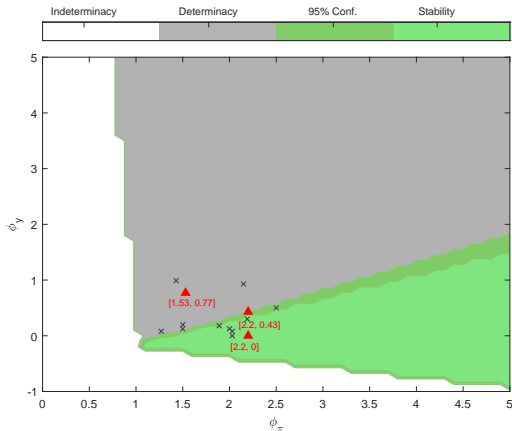
Note: Simulated determinacy (gray) and stability (green) region. Gray 'x' marks correspond to other empirically

found Taylor rule coefficients in the literature.

Anchored vs. de-anchored expectations

Global solution doesn't exists outside of the stability region

| | $\phi_\pi = 2.5,\ \phi_Y = 0$ | | | $\phi_\pi = 2.2,\ \phi_Y = 0.43$ | | $\phi_\pi = 1.53,\ \phi_Y = 0.77$ | |
	1st order	3rd order	global	1st order	3rd order	1st order	3rd order
Moments of macroeconomic variables							
std(C)	2.73	2.74	2.74	2.43	4.47	2.16	16.37
std(N)	0.93	0.94	0.94	1.21	2.20	1.47	11.89
std(i)	1.76	1.76	1.76	3.72	6.23	5.55	39.54
std(π)	0.70	0.70	0.70	2.89	5.01	4.94	36.12
Behavior of price dispersion and implied output loss, $\Delta_t = 100\left(1 - S^{-1}\right)$							
mean(Δ)	-0.00	0.05	0.05	-0.00	0.87	-0.00	1.85
std(Δ)	0.00	0.06	0.06	0.00	2.11	0.00	14.02
max(Δ)	0.00	0.42	0.43	0.00	16.85	0.00	67.47
min(Δ)	-0.00	0.00	0.00	-0.00	-7.76	-0.00	-143.34
$\pi_t > \pi^{upper}$	0.00	0.00	0.00	0.00	0.67	0.00	20.35
$S < 1$	0.00	0.00	0.00	0.00	12.63	0.00	44.98

Misalignment in relative prices
Our results is specific to Calvo pricing

Calvo fairy

- Calvo fairy is in old fairytale from Vienna. Each morning when Vienna firms wake up, an exogenous fraction are visited by the Calvo fairy, a benevolent spirit who lives in MuseumsQuartier . After the visit they have the privilege to change their price.
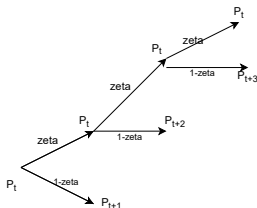
Non-linear solution

1. preserves distribution of prices across product varieties and break certainty equivalence,
2. misalignment in relative prices can start **price-inflation spiral**
3. Optimal price markup
   - PV of Marginal Revenues = MARKUP * PV of Marginal Costs
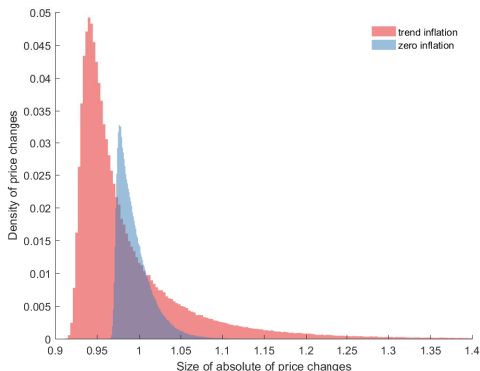
Trend inflation

How to anchor inflation expectations in the high inflation periods like we face now?
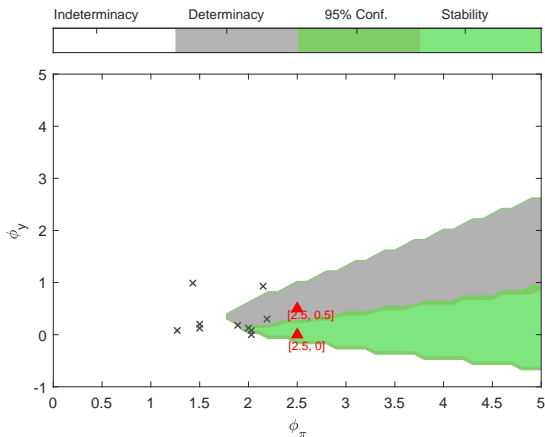
Note: Simulated determinacy (gray) and stability (green) region. Gray 'x' marks correspond to other empirically found Taylor rule coefficients in the literature.

Contents

Macro-finance Model

- ▶ price-dispersion spiral is relevant and quantitatively pronounced in widely used modeling framework of term structure of interest rates
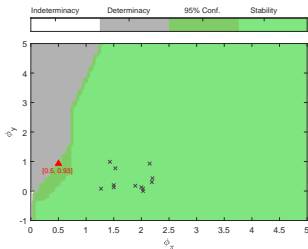
- ▶ Rudebush and Swanson (2012)

Figure: Approximation at $\pi = 0\%$.
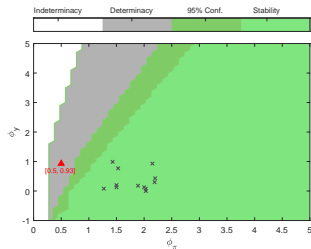
Figure: Approximation at $\pi = 4\%$.

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Concluding remarks

- price dispersion driven inflation spiral provides a new challenge for the central bank to anchor inflation expectations.
- we introduce concept of the stability region as a non-linear counterpart to the determinacy region
- From a modeling perspective the main implication: moving to and from linear and non-linear models is not simple or straightforward.
- The presence of a price-inflation spiral in non-linear models requires careful re-calibration and tuning of Taylor rule parameters

Contents

No-existence of global solution

- we look for the (nonlinear) policy function $S_t = h(S_{t-1}, A_t)$ as the fixed point of the nonlinear system of difference equations implied by the DSGE model.
- problem: for some value of (low) A_t (that implies high Π_t), the relevant state space in the S_{t-1}-dimension becomes unbounded $<==>$ no specified S_{t-1} grid is large enough, S_t always falls outside of that grid (because $S_t > S_{t-1}$ when in the instability region $\Pi_t > \Pi^{threshold}$)
- enlarging the S_{t-1}-grid further is of no help; the new S_t will again fall outside the enlarged S_{t-1}-grid

No-existence of global solution

The state variable S_t increases the solution space at faster rate

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$$
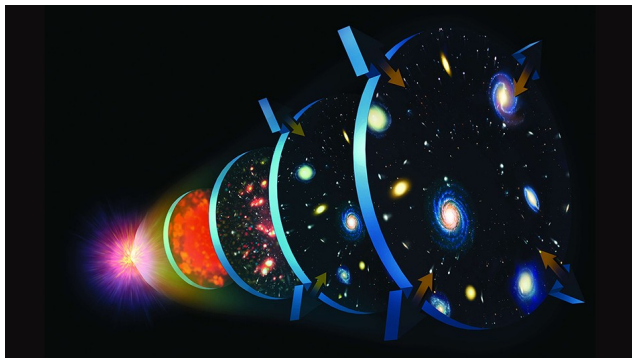
Note: Expanding universe.

Departure: standard NK model

Households

$$U(C_t, N_t) = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} \right], \quad \varphi, \chi > 0, \tag{4}$$

subject to the flow budget constraint:

$$B_t + P_t C_t = W_t N_t + D_t + R_{t-1} B_{t-1} - \tau_t. \tag{5}$$

Departure: standard NK model

Firms

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} \left[P_t(i) Y_{t+k}(i) - P_{t+k} W_{t+k} N_{t+k}(i) \right] \right\}, \quad (6)$$

$$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \Upsilon_{t+k} MC_{t+k}, \quad (7)$$

where $\Upsilon_{t+k} = \frac{\zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}} Y_{t+k}}{\sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\epsilon-1} Y_{t+k}}$,

Monetary Policy

$$\log(i_t) = \log(\bar{\imath}) + \phi_\pi [\log(\Pi_t) - \log(\overline{\Pi})] + \phi_Y \log\left(\frac{Y_t}{\overline{Y}} \right), \quad (8)$$

Price dispersion

Firms produce goods,

$$Y_t(i) = A_t \bar{K}^\theta (N_t(i))^{1-\theta}, \tag{9}$$

To sell their output, firms face downward sloping demand curve for their goods,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t. \tag{10}$$

Workers are all the same and the aggregation of hours worked is $N_t = \int_0^1 N_t(i) di$. Aggregation thus delivers,

$N_t = \left(\frac{Y_t}{A_t K^\theta} \right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\theta}} di$

which can be re-written as

$$Y_t = S_t^{-1} A_t \bar{K}^\theta N_t^{1-\theta}, \tag{11}$$

where variable $S_t^{\frac{1}{1-\theta}} = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} di$ defines price dispersion.

S_t^{-1} measures the costs of misalignment in relative prices.

Price dispersion

Aggregation

We can use the Calvo (1993) result and rewrite $S_t^{\frac{1}{1-\theta}}$ recursively as

$$
\begin{aligned}
S_t^{\frac{1}{1-\theta}} &\equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon}{1-\theta}} di, \\
&= (1-\zeta)(p_t^*)^{\frac{-\varepsilon}{1-\theta}} + \zeta (\Pi_t)^{\frac{\varepsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}.
\end{aligned}
\tag{12}
$$

Note that this can be done because of the assumption of the exogenous probability of price adjustments in Calvo.

$$
S_t^{\frac{1}{1-\theta}} = (1-\zeta) \left[\frac{1 - \zeta (\Pi_t)^{\epsilon-1}}{1-\zeta} \right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}.
\tag{13}
$$

- ▶ upper and lower bound on inflation

Price dispersion spiral

Definition
Inflation for which,

$$S_t > S_{t-1} \qquad (14)$$

- ▶ if prices are widely dispersed in the economy the dispersion will become self-reinforcing

- ▶ the higher the steady state of inflation the higher is the probability that exogenous inflationary shocks will trigger this price-inflation spiral

- ▶ Trend inflation spreads out the distribution of prices as those firms which cannot change their low prices are left behind further and further from the optimal price as the price level grows. When these firms can finally change their price they create large inflation

Model Solution Methods

- traditionally macroeconomic dynamics have been studied primarily in terms of (log-)linear approximations to the true model solution
  - no price dispersion: linear model approximate price distribution by a line
  - uncertainty about future enters the decision function of agents because of certainty equivalence
    - I might not buy a house today even if I am having good times because I am uncertain about future development
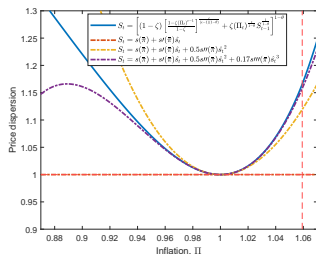
Model Solution Methods
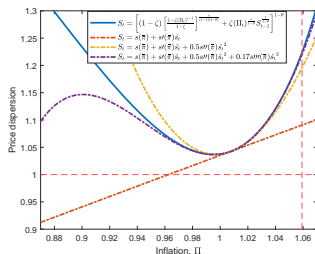

Figure: Approximation at $\pi = 0\%$.

Figure: Approximation at $\pi = 4\%$.

Blue solid line, S_t, represents price dispersion as a function of Π_t, as given by equation (13), at point $S_{t-1} = \bar{S}$. Dash-dotted lines are first (red), second (yellow) and third (purple) order approximations of S_t at $\pi = 0\%$ and at $\pi = 4\%$. Light red Horizontal and vertical dashed lines represent the lower bound on S_t and upper bound on Π_t.

Real rigidity

Calvo not enough to capture effect of price stickiness on quantities. Real rigidities are the mechanism which lowers firms incentive to increase prices in the face of a rise in nominal demand.