Optimal Capital Controls and Real Exchange Rate Policies:

Policies:

A Pecuniary Externality Perspective[*]

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Capital Controls, Real Exchange Rate Policy, Financial Frictions, Financial Crises, Financial Stability, Optimal Taxation, Macro-Prudential Policies.

E52, F37, F41.

Abstract

A new theoretical literature studies the use of capital controls to prevent financial crises in models in which pecuniary externalities justify government intervention. Within the same theoretical framework, we show that when ex-post policies such as defending the exchange rate can contain or resolve financial crises, there is no need to intervene ex-ante with capital controls. On the other hand, if crises management policies entail some efficiency costs, then crises prevention policies become part of the optimal policy mix. In the standard model economy used in the literature with costly crisis management policies, the optimal policy mix combines capital controls in tranquil times with support for the real exchange rate to limit its depreciation during crises times. The optimal policy mix yields more borrowing and consumption, a lower probability of financial crisis, and twice as large welfare gains than in the socially planned equilibrium with capital controls alone.

JEL Classification: E52, F37, F41

Keywords: Capital Controls, Exchange Rate Policy, Financial Frictions, Financial Crises, Optimal Taxation, Macro-Prudential Policies.

1 Introduction

In response to the global financial crisis and its costly aftermath, a new policy paradigm emerged in which old fashioned government policies such as capital controls and other restrictions on credit flows became part of the standard crisis prevention policy toolkit (the so called macro-prudential policies). A few, large emerging market economies experimented with these tools. And even the traditionally conservative IMF changed its orthodox views on capital controls, advocating the use of such measures when other tools are not available or have run their course of action—see Blanchard and Ostry (2012) and IMF (2012).

The key rationale underpinning the use of capital controls is financial stability.[1], [2] The financial stability motive is the focus of the influential contributions of Korinek (2010) and Bianchi (2011).[3] Their analysis is based on variants of a common theoretical framework proposed by Mendoza (2002, 2010). In this framework, the scope for policy intervention arises because of a pecuniary externality stemming from the presence of a key relative price in the collateral constraint faced by private agents. In this environment, prudential interventions (i.e. before a financial crisis occurs) may be desirable because they can make agents internalize the consequences of this externality on their individual decisions. Capital controls in this setting can discourage financial excesses, reduce the amount that agents borrow, thereby lowering the probability of a financial crisis, and hence enhance welfare.

In this paper, we provide an integrated analysis of alternative policy tools that can be interpreted in terms of fiscal, monetary and macroprudential policies using the same model economy studied by Bianchi (2011). We find that, when financial stability is the sole motive for policy intervention, the optimal policy design aims at supporting the value of the collateral and hence the agents' borrowing capacity during crises times. In this context, policies that support the real exchange rate (or more generally collateral price support policies) during a financial crisis dominates by a large margin, from a welfare point of view, prudential controls on capital flows. The dominance of price support policies relies (perhaps unrealistically) on

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[1]Blanchard and Ostry (2012) make explicit reference to the pecuniary externality perspective when motivating the IMF's view on the use of capital controls: "*If there are external effects from foreign borrowing (think of amplified crisis risks for the country, where the risks are not internalized by the borrower), then capital controls can act as Pigouvian taxes and constitute an optimal response at the country level, helping agents to internalize the external effects of their borrowing*".

[2]Historically, as documented by Magud, Reinhart, and Rogoff (2011), capital controls have been adopted for fear of capital flows reversal, fear of excessive risk taking, and to contain asset price bubbles. Other traditional reasons include concerns for competitiveness and monetary policy independence—see more on these below.

[3]See also Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2012, 2013) and Benigno et al. (2013).

the assumption that they are costless to use. Indeed, when we assume that supporting the value of collateral during crisis times is costly, it becomes optimal to combine price support policies with macroprudential policies such as capital controls.

In our analysis, then, the rationale for macroprudential policies relies on the extent to which price support policies are cost-effective rather than the amount that agents borrow in the unregulated economy during tranquil times. This novel element of our analysis emphasizes the interaction between ex-ante (normal times) and ex-post (crises times) policy interventions: when price support policy is fully effective in crises times (i.e. it is able to address the pecuniary externality distortion at no other cost) there is no scope for ex-ante policy intervention. However, if the policy is costly in crises times, it is optimal to adopt capital controls during normal times as a way to limit the occurrence of the crises, combined with price support policies in crises times to mitigate their severity. We find that the optimal combination of ex ante and ex post policy interventions achieves welfare gains of 1.10% of tradable consumption relative to the unregulated economy, which is much higher than the typical value found in the literature.

As the vehicle to convey our messages, we adopt the same model economy as in Bianchi (2011). This is a two-sector (tradables and nontradables) small open, endowment economy with an occasionally binding international borrowing constraint. Quantitatively, this model has been successful in reproducing the business cycle and the crisis dynamics properties of a typical emerging market economy. In this class of models, a financial crisis event (also labelled a Sudden Stop in capital or credit flows) occurs when the constraint binds. In our model, the value of total current income generated both in the tradable and nontradeable sectors limits borrowing, denominated in units of tradable consumption. When the borrowing constraint binds, the decline in the relative price of nontradables generates a balance sheet effect and leads to a Fisherian debt-deflation spiral.

In this economy there is a well defined scope for government intervention, but there are multiple instruments or tools with which policy could be conducted. The pecuniary externality arises from the fact that individual agents do not internalize the aggregate effect of their borrowing decisions on the relative price of nontradable goods, which is the price that enters the collateral constraint. There are three type of taxes that can be used to correct it: a tax/subsidy on foreign debt or a tax/subsidy on tradable consumption and a tax/subsidy on nontradable consumption. The tax on foreign debt is usually interpreted as a capital control, while taxes on either tradable or nontradable consumption can be interpreted as a real exchange rate interventions because they affect directly the relative price of nontradables.[4]

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[4]The interpretation of the relative price of nontradables as the real exchange rate is standard in the

In our policy analysis, we consider all three instruments and study their relative effectiveness in welfare terms. Differently from the existing literature, to conduct the policy analysis we follow a Ramsey optimal taxation approach, assuming that the government budget is always balanced.

We first study the Ramsey problem when capital controls are the only policy tool available, and the government budget constraint is balanced through lump-sum transfers/taxes. Consistent with Bianchi (2011) and Korinek (2010), we find that, in this case, it is Ramsey optimal to limit the amount that agents borrow in normal times, while no action is needed during crises times. The reason why capital controls are not used by the Ramsey planner in crisis times is that, in this model, they cannot affect the allocation when a crisis occurs (i.e. when the borrowing constraint binds). Thus, in this setting, when capital controls are the only policy tool available, the best that the government can do is to reduce the probability that a crisis occurs by inducing the private sector to borrow less than in the decentralized equilibrium.

Next we show that a policy of supporting the real exchange rate during crisis times by relaxing the borrowing constraint when it binds, can achieve a much higher level of welfare. In fact, we show that such a policy can undo the borrowing constraint completely and, as a result, support an equilibrium in which agents behave as if they were in an unconstrained allocation. Importantly, as we shall see, this policy is time-consistent. The policy can be implemented with a subsidy on non tradable consumption or a tax on tradable consumption. The result hinges on the ability of the Ramsey planner to manipulate the value of collateral with consumption taxes or subsidies that affect the relative price of nontradable goods without creating any other distortions. Indeed, since these consumption taxes or subsidies are rebated or financed through lump-sum transfers or taxes, they are costless and do not entail further distortions.

Finally we show that, when lump-sum transfers/taxes are no longer available, price support policies during crisis times become costly, and capital controls in normal times complement exchange rate policy in crises times under the optimal policy mix. When ex-post policies are costless they can be used all the way to remove the borrowing constraint, and there is no need to engage in ex-ante policy interventions such as capital controls. But when the use of ex-post policies entails efficiency losses or costs (such as when there are other policy objectives to be traded off for financial stability), then ex-ante policy interventions

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literature. See for instance Bianchi (2011), Caballero and Lorenzoni (2014), Mendoza (2002), Korinek (2010), Jeanne (2012). Alternatively, the consumption taxes (subsidies) could be interpreted more literally as domestic fiscal policy tools.

like prudential capital controls are called for to maximize welfare. Notice that this rationale for the use of ex-ante policy interventions is not related to the amount that agents borrow in the competitive equilibrium of the economy without government intervention. Under the optimal policy with both instruments, there is more borrowing, a lower probability of crisis, and as a result much higher welfare, with gains of 1.10% of tradable consumption relative to the unregulated economy as compared to only .41% with capital controls alone.

The paper relates to a few other recent contributions in the literature on pecuniary externalities which focused both on ex ante and ex post policies. Benigno et al. (2013) analyze the extent to which private agents overborrow or underborrow in a production version of our economy. They show that the allocation chosen by a social planner away from the crisis depends on the planner's ability to mitigate a crisis, should one occur. Benigno et al. (2013) do not analyze any implementation issues or optimal policy problems. Jeanne and Korinek (2013) study the time-consistent mix of ex-ante macroprudential regulation and ex-post bailout transfers in a three-period economy in which the relative price that enters the borrowing constraint is an asset price. The presence of the asset price in the policy problem opens the door to a time-consistency issue, which is their main focus and it is not present in our model. Jeanne and Korinek (2013) also study the role of ex-ante and ex-post policies in their model, but they restrict the set of policy tools along two dimensions: first they restrict the use of distortionary taxation to the contingency in which the constraint binds, while we allow the policy maker to choose freely which instrument to use both in normal and crisis times; and second, they do not consider all the possible tools in the context of their model.

Other new theoretical approaches rationalized the use of capital controls. One approach motivates the use of capital controls with the possibility of manipulating the intra or intertemporal terms of trade—conceptually analogous to the use of tariffs to manipulate the goods' terms of trade (Costinot, Lorenzoni and Werning (2014) and De Paoli and Lipinska (2013). Schmitt-Grohe and Uribe (2015) examine the role of capital controls in an economy with downward nominal wage rigidity and a fixed exchange rate regime. They focus on competitiveness issues and are silent on the financial stability motive we focus on. Fahri and Werning (2012) study capital controls as a way to address the impossibility to simultaneously have an open capital account, a fixed exchange regime, and an independent monetary policy (as known as the "impossible trilemma"). Likewise, Devereux and Yetman (2014) analyze capital controls as a way to restore monetary policy effectiveness when the nominal interest rate reaches the zero lower bound in a global liquidity trap context.

Other approaches have focused on the role of capital controls when there are multiple distortions or objectives. For instance, Brunnermeir and Sannikov (2014) show that restrictions

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to capital flows can be welfare improving in an economy with multiple goods, incomplete financial markets, and inefficient production, but does not discuss issues of optimal mix between ex-ante and ex-post interventions. Ottonello (2015) studies optimal exchange rate policy with downward nominal wage rigidity, flexible exchange rates, and a borrowing constraint like the one in our model. His analysis focuses on a restricted set of instruments similar to Jeanne and Korinek (2013) and discusses the trade-offs that exchange rate policy faces between competitiveness and financial stability considerations.

More broadly, our paper shares the emphasis on price support policies that limit the depreciation of the real exchange rate during crisis times with the work of Chang, Cespedes and Velasco (2012), who examine the role of other unconventional policy tools such as credit policies and direct interventions in the foreign exchange market. While they study more realistic forms of government intervention, they do not compute optimal policy, but rather focus on the transmission mechanism of alternative policy tools.[5] In an different framework, Martin and Ventura (2014) also suggest policies that relax the collateral constraint by properly managing the size of "bubbles". More generally, our study of alternative policy tools is related to the work by Correia, Nicolini and Teles (2008) in which the role of price stickiness for the design of monetary policy depends on the existence of alternative fiscal policy tools.

Finally, in terms of the solution techniques, we apply the same algorithm proposed by Benigno et al. (2012) to solve numerically for the Markov Perfect optimal policy problem in the context of a production version of our economy in which a time-consistency issue arises.

The rest of the paper is organized as follows. Section 2 describes the model environment, the scope for government intervention, and the alternative government instruments that we consider. Section 3 studies optimal capital control policy. Section 4 analyzes optimal real exchange rate policy. Section 5 considers the joint use of capital controls and real exchange rate policies when lump-sum transfers/taxes are not available, as well as some robustness analysis. Section 6 relates the main results of the paper to countries's experience with capital controls and price support policies over the past 20 years or so. Section 7 concludes. The numerical solution methods we use as well as other technical material including proofs and extensions are reported in an appendix for online publication.

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[5]In an optimizing neoclassical framework without credit frictions, Calvo, Reinhart and Vegh (1995) also analyze the role of real exchange rate targeting as a temporary stabilization policy.

2 The model environment

In this section we describe our model economy and discuss its key assumptions. We then characterize the competitive equilibrium of the model that we examine. Next we identify the externality that gives rise to scope for government intervention. And finally, we discuss the alternative government policy instruments that we will analyze in the rest of the paper.

We consider a small open economy in which there is a continuum of households $j \in [0, 1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t u \left(C_t^j \right) \right\}, \tag{1}$$

where C_t^j is the consumption basket for an individual j at time t, and is β the subjective discount factor. E_0 denotes the conditional expectation at time 0. We assume that the period utility function is isoelastic:

$$u \left(C_t^j \right) \equiv \frac{1}{1 - \rho} \left(C_t^j \right)^{1-\rho}.$$

The consumption basket, C_t, is a CES aggregate of tradable and nontradable goods (omitting the subscript j to simplify notation):

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{\frac{\kappa-1}{\kappa}} + (1 - \omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \tag{2}$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of the two goods in the utility function.

We normalize the price of tradable goods to 1 and denote the relative price of the non-tradable goods with P^N. The aggregate price index is then given by

$$P_t = \left[\omega + (1 - \omega) \left(P_t^N \right)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}.$$

Here we note that there is a one-to-one link between the aggregate price index P and the relative price P^N.

Households maximize utility subject to their budget constraint, which is expressed in units of tradable consumption, and a borrowing constraint. The asset menu includes only a one-period bond denominated in units of tradable consumption. Each household has two stochastic endowment streams of tradable and non-tradable output, $\{Y_t^T\}$ and $\{Y_t^N\}$. For simplicity, we assume that both $\{Y_t^T\}$ and $\{Y_t^N\}$ are Markov processes with finite, strictly

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positive support. Therefore the current state of the economy can be completely characterized by the triplet $\{B_t, Y_t^T, Y_t^N\}$. Thus, the budget constraint each household faces is

$$C_t^T + P_t^N C_t^N + B_{t+1} = Y_t^T + P_t^N Y_t^N + (1+r) B_t, \qquad (3)$$

where B_{t+1} denotes the bond holding at the end of period t, and $(1+r)$ is the given world gross interest rate.

Access to international financial markets is not only incomplete but also imperfect in the sense that, by assumption, the amount that each individual can borrow is limited by a multiple of his current total income:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N \right]. \qquad (4)$$

One justification of (4) is in terms of liquidity constraints. By this interpretation, lenders require households to finance a fraction ϕ of their current expenses—which include consumption, debt repayments and taxes—out of current income (see Mendoza (2002) for this interpretation):

$$\phi \left(Y_t^T + P_t^N Y_t^N \right) \geq C_t^T + P_t^N C_t^N - (1+r)B_t. \qquad (5)$$

In fact, by combining (5) with (3) we obtain (4). Another justification of (4) appeals to an environment in which the borrower engages in fraudulent activities during the period in which the debt is contracted and prevents creditors from seizing any future income (see Bianchi (2011) for this interpretation). Note here that (5) depends on pre-tax income rather than post-tax income. So, in an environment with default, the individual who defaults is left with her/his full tax-obligation.[6]

At the empirical level, as Mendoza (2002) and Bianchi (2011) emphasize, a specification in terms of current income is consistent with evidence on the determinants of access to credit markets, on lending criteria and guidelines used in mortgage and consumer financing (e.g., Jappelli 1990, Jappelli and Pagano, 1989). The assumption that nontradable goods can be pledged as collateral is consistent with the evidence reported by Tornell and Westermann (2005) on the use of international credit to finance booms in the nontradable sector.

The key feature of (4) is that it captures currency mismatches in the balance sheet of the economy—see Krugman (1999). In fact borrowing is denominated in units of tradable consumption, while both the tradable and the nontradable endowment can be pledged as

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[6]See section 5 for a discussion of what happens when households can default on their tax obligation as well, and the borrowing constraint depends on post-tax income.

collateral. Indeed, currency mismatches have been one of the main vulnerability of emerging market economies in the numerous financial crises of the 1990s and the 2000s and continued to be a crucial policy challenge in the post global financial crisis period—see for instance Cespedes, Chang and Velasco (2004, 2012), Shin (2013), and Acharya et al. (2015) for a discussion.

From a model perspective, a financial crisis occurs when the constraint binds and the model dynamics changes nonlinearly; an event that is endogenous in the model. Yet the long-run business cycle properties of the economy are only marginally affected by the crises events (Mendoza, 2010). A unique feature of this model environment, therefore, is to nest endogenous financial crisis dynamics triggered by small exogenous disturbances within regular business cycles.

In our small open economy, the motive for borrowing arises from the assumption that $\beta(1+r) < 1$ so that agents are impatient compared to foreign lenders. However, we assume that there is a lower bound on debt strictly greater than the natural debt limit, $\underline{B} > B^n$, such that $B_t \geqslant \underline{B}$, for all t.[7] This lower bound guarantees that the competitive equilibrium of the economy without government intervention and without the international borrowing constraint (4) is well defined. In particular, it guarantees that there is an ergodic distribution of debt with finite support, and both tradable and nontradable consumption have a strictly positive lower bound, while the nontradable price also has finite support with strictly positive lower bound. In order to focus on non-trivial policies, we also assume that, given Y_t^T and Y_t^N, when $B_t = \underline{B}$, the competitive equilibrium allocation always violates the borrowing constraint (4).[8]

As we discuss in the online appendix, we finally note that our calibration, and in particular the assumption that tradable and nontradable goods are complement ($\kappa < 1$), rules out the possibility of multiple equilibria.

2.1 Competitive equilibrium

We provide a full characterization of the competitive equilibrium of the economy with the borrowing constraint and no government intervention in the online appendix. In this equilibrium, households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T and B_{t+1}. The

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[7]If C^T and C^N are strong substitutes, this constraint may bind; since the evidence is that C^T and C^N are complements, we can ignore this possibility.

[8]This restriction amounts to a lower bound on ϕ.

intratemporal allocation between tradeable and non tradeable goods is given by

$$\frac{(1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}}} = P_t^N. \tag{6}$$

Goods market clearing for tradeable goods yields

$$C_t^T = Y_t^T - B_{t+1} + (1+r) B_t, \tag{7}$$

while for non tradeable goods we have:

$$C_t^N = Y_t^N. \tag{8}$$

The quantitative properties of this equilibrium are well known (see Mendoza (2002) and Bianchi (2011)). Here, it is important to note only that, as Bianchi (2011) illustrated, this very same model can account well for key business cycles statistics as well as the incidence and severity of financial crises in a typical emerging market economy like Argentina. Throughout the paper, therefore, whenever we need to resort to numerical methods, for illustrative purposes, we set all parameter values of the model exactly like in Bianchi (2011)—and a summary table is reported in the online appendix.

2.1.1 Unconstrained Equilibrium

As we shall see below, two of the government policy instruments that we consider, when used optimally, can completely remove the effects of the constraint (4) and achieve an allocation that is identical to the competitive equilibrium of the model without the borrowing constraint (4). In what follows we refer to this allocation as the "unconstrained equilibrium" (UE) and we characterize it in the online appendix.[9]

In the deterministic steady state of the model, since agents are impatient, the allocation will tend to converge towards the natural debt limit. [10] In our stochastic economy, agents engage in precautionary saving so that the probability of hitting the natural debt limit is zero.

Also note here that the unconstrained equilibrium characterizes an allocation in which financial markets are incomplete so that there are inefficient variations in consumption due

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[9] As we discussed above, the existence of a lower bound on debt which is strictly greater than the natural debt limit guarantees that the competitive allocation without borrowing constraint has an ergodic distribution of debt with finite support under the assumption that $\beta(1+r) < 1$.

[10] In our model, this level equals (minus) the annuity value of the lowest tradable endowment value.

to the lack of state contingent debt. For completeness, in the online appendix, we describe the first best allocation in which agents in the small open economy have access to state contingent securities and compare it with the unconstrained allocation.

2.2   Pecuniary externality

In order to understand the rationale for policy intervention in our model, we first follow the recent literature—e.g., Lorenzoni (2008), Korinek (2010) and Bianchi (2011)—and focus on a benevolent social planner problem with restricted planning abilities. In the rest of the paper, we then focus on Ramsey optimal policy. In particular, we initially assume that the social planner can directly choose the level of debt subject to the credit constraint while allowing goods markets to clear competitively. Unlike the representative agent in the competitive equilibrium of the model, the social planner internalizes the effects of his/her borrowing decisions on the equilibrium relative price of nontradables. This is relevant in our set up because, when the constraint binds, the agents' borrowing capacity depends on the value of the collateral, which in turn is determined endogenously by the equilibrium relative price of nontradables.

2.2.1   Social planning problem

Specifically, the benevolent social planner maximizes (1) subject to the same borrowing constraint (4) that private agents face and the market clearing conditions for tradables and nontradables goods (7) and (8). In specifying this problem, the equilibrium price of nontradables is determined competitively according to the pricing rule (6). This condition also serves as a constraint on the planning problem to eliminate P_t^N from the borrowing constraint.[11]

Figure 1 illustrates graphically the consequences of the presence of the borrowing constraint by comparing the policy functions of the endogenous variables (C_t^T, B_{t+1}, P_t^N) for a negative one-standard deviation shock.[12] We consider the three allocations we defined above: the competitive equilibrium with borrowing constraint (CE), the social planner problem (SP)

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[11]This formulation is usually referred to as "constrained-efficient" planning problem in the literature. A second possibility, sometimes referred to as the "conditionally-efficient" problem, is to determine this relative price by imposing as a constraint on the problem the competitive equilibrium *policy function* (in our case $P_t^N = f^{CE}(B_t, Y_t^N, Y_t^T)$). In our endowment economy, these two definitions give exactly the same result and do not affect the normative analysis. See Kehoe and Levine (1993) and Lorenzoni (2008) for more details and a discussion.

[12]A policy function is the non-linear equilibrium relation between the endogenous variables of the model and its exogenous and endogenous states (in our case, the triplet $\{B_t, Y_t^N, Y_t^T\}$).

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and the unconstrained equilibrium (UE).[13]

Figure 1 clearly shows the difference between the policy functions of the constrained (CE and SP) and the unconstrained (UE) equilibrium allocations. In particular the UE allocation features a much higher level of tradable consumption and debt, as well as a higher relative price of nontradable goods, compared to the CE and SP allocations. In the absence of the borrowing constraint, agents can borrow freely from international capital markets to smooth consumption for any given stock of existing debt. In contrast, the CE and SP allocations are relatively close: they diverge only slightly in the region in which the constraint is not binding, but it is expected to bind in the future; otherwise they coincide exactly including particularly in the region in which the constraint binds.

In the CE and the SP allocations, in the region in which the constraint binds (i.e., when there is a financial crisis in our model), both consumption of tradables and the relative price of nontradables fall sharply.[14] This decline is the consequence of the so-called "Fisherian deflation" or fire sale mechanism emphasized in the financial crisis literature. When borrowing is constrained, consumption is much lower relative to the desired amount in the unconstrained equilibrium. Lower tradable consumption is accompanied by a decline in relative price of nontradables, which in turn reduces the value of collateral, tightening borrowing capacity and reducing tradable consumption further, a feedback loop results in even lower relative price of nontradables and tradable consumption.

As emphasized by Lorenzoni (2008), Korinek (2010) and Bianchi (2011), when the constraint does not bind (i.e. in normal times), but it is expected to bind in the future with some positive probability, agents in the competitive equilibrium consume more than in the social planner allocation. As we show in appendix, this difference arises because individual agents don't take into account the additional benefit of reducing consumption today, captured by the term $E_t \left(\lambda_{t+1}^{SP} \Sigma_{t+1}^{SP} \right)$ in the planning problem, which in turn represents the marginal benefit of consuming more when the constraint binds in the future.

Note however that, in this endowment economy, for a given state $\left\{ B_t, Y_t^N, Y_t^T \right\}$ in which the constraint binds, the CE and the SP allocations coincide. For a given amount of existing debt, tradable consumption will be the same in the two allocations since it is constrained by the borrowing limit. The equilibrium relative price of nontradables is also equalized, since the consumption of nontradables is pinned down by its endowment.[15]

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[13]Complete solutions for these allocations have to be computed numerically, and we use the global solution methods that we describe in appendix.

[14]In the figure, the binding region starts in correspondence to the kink in the policy functions.

[15]Recall that the relative price of nontradables is proportional to the ratio of tradable over nontradable consumption.

2.3 Alternative policy instruments

While there is a well defined scope for government intervention, in this economy, there is a variety of instruments or tools with which policy could be conducted. In fact, in our model economy, there are at least three types of taxes that can be used: a tax/subsidy on debt, a tax/subsidy on tradable consumption and a tax/subsidy on nontradable consumption. In our policy analysis, in the rest of the paper, we consider all of them, studying their relative effectiveness in welfare terms as well as their joint use.[16]

To conduct the policy analysis we take a Ramsey optimal taxation approach assuming that the government budget is always balanced. For given policy instrument(s), the Ramsey planner maximizes the representative household's utility function, subject to the resources constraints and the first order conditions of its maximization problem.

Tax on debt The first policy tool that we examine is a tax τ_t^B (< 0) or a subsidy (> 0) on one-period debt issued at time t, B_{t+1}. This instrument is usually referred to as a capital control.[17] When we allow for lump-sum transfers/taxation, the government budget constraint is:

$$T_t = \tau_t^B B_{t+1}, \tag{9}$$

where T_t denotes the lump sum transfer or tax. In this case, the household's budget constraint in the competitive equilibrium of the model becomes

$$C_t^T + P_t^N C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau_t^B) + (1 + r) B_t, \tag{10}$$

while the liquidity constraint becomes

$$\phi \left(Y_t^T + P_t^N Y_t^N \right) \geq C_t^T + P_t^N C_t^N - (1 + r)B_t + T_t. \tag{11}$$

Combining these three constraints, gives rise to the same international borrowing constraint as before, so that access to international financial market continues to be constrained by (4).

Taxes on consumption The other two policy tools that we study are consumption taxes on non-tradable and on tradable goods. Both policy tools influence directly equation (6) and

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[16]Notice here that any given allocation could be implemented by different tax instruments. This indeterminacy in the tax instruments depends on the fact that there are two decisions margins but we are considering three possible policy tools—see also Costinot et al. (2014) on this.

[17]One of the best known cases of a use of such a tool is the Brazilian IOF tax. See Pereira and Harris (2012) for a detailed account of this actively researched country case.

$_2$ affect the relative price of nontradable goods, P_t^N, which in the context of our economy is a
$_3$ proxy for the real exchange rate—see for example Mendoza (2002), Caballero and Lorenzoni
$_4$ (2014), Korinek (2010), Bianchi (2011), Jeanne (2012), and Schmitt-Grohe and Uribe (2015)
$_5$ for the same interpretation. For this reason, in what follows, we refer to these two tools as
$_6$ "real exchange rate policy" or "exchange rate policy" for brevity. Alternatively these taxes
$_7$ can be also interpreted as fiscal devaluation/revaluation when monetary policy tools are not
$_8$ available (i.e. in a fixed exchange rate regime or in a currency union).

$_9$ With a tax on nontradable consumption, $(1 + \tau_t^N)$, the household's budget constraint
$_{10}$ becomes

$$C_t^T + P_t^N(1 + \tau_t^N)C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + r) B_t, \tag{12}$$

$_{11}$ where $\tau_t^N > (<) \, 0$ is now a tax (or a subsidy) on nontradable consumption and $T_t > (<) \, 0$
$_{12}$ is a government lump-sum transfer (or tax). As in the case of capital controls, we assume
$_{13}$ that the government budget balances period by period:

$$T_t = \tau_t^N P_t^N C_t^N. \tag{13}$$

$_{14}$ In this case, the liquidity constraint becomes

$$\phi \left(Y_t^T + P_t^N Y_t^N\right) \geq C_t^T + (1 + \tau_t^N)P_t^N C_t^N - (1 + r)B_t + T_t, \tag{14}$$

$_{15}$ which combined with the individual and the government budget constraints above determines
$_{16}$ the same international borrowing constraint as before (4).

$_{17}$ With a tax on tradable consumption, $(1 + \tau_t^T)$, the household now faces the following
$_{18}$ budget constraint:

$$(1 + \tau_t^T)C_t^T + P_t^T C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + r)B_t. \tag{15}$$

$_{19}$ The government budget constraint continues to balance period by period:

$$T_t = \tau_t^T C_t^T, \tag{16}$$

$_{20}$ and the borrowing constraint remains as in (4).

3 Optimal capital controls

1 We now study the optimal Ramsey problem when the policy tool is τ_t^B. The Ramsey problem
2 for τ_t^B is to choose a competitive equilibrium that maximizes (1). More formally:

Definition 1 *For a given $\{B_0\}$ and assuming that $\{Y_t^T\}$ and $\{Y_t^N\}$ are Markov processes*
with finite, strictly positive support, the Ramsey problem for τ_t^B is to choose a competitive
equilibrium that maximizes

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t u\left(C_t\right) \right\},$$

3 *subject to the resource constraints (7) and (8), the government budget constraint (9), the*
4 *borrowing constraint*

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N \right],$$ (17)

5 *and the first order conditions of the household,*

$$u'(C_t)C_{C^T}(1+\tau^B) = \lambda_t + \beta\left(1+r\right)E_t\left[u'(C_{t+1})C_{C^T}\right],$$ (18)

6

$$\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}} = P_t^N.$$ (19)

7 We can now state the following proposition that qualifies the main result of Bianchi
8 (2011).

9 **Proposition 1** *In an economy defined by (1), (4), (12) and (9) with a tax on τ_t^B as*
10 *the government instrument, the Ramsey optimal policy with τ^B as instrument replicates the*
11 *social planner allocation (SP). Moreover the optimal policy is time-consistent.*

12 PROOF: see online appendix.

13 A few remarks are in order here. From a policy perspective, as discussed by Bianchi
14 (2011) and noted in the previous section, when the constraint binds (i.e. $\lambda_t^{SP} > 0$), the social
15 planner allocation coincides with the competitive equilibrium allocation, and therefore it is
16 optimal to set $\tau_t^B = 0$. When the constraint does not bind, but it can bind with positive
17 probability in the next period (i.e. $\lambda_t^{SP} = 0$, but $E_t[\lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}] > 0$ in equation (44) in the
18 online appendix), the optimal state contingent τ_t^B is a tax on borrowing ($\tau_t^B < 0$). Thus,
19 it is optimal to engage in a policy intervention even when the constraint does not bind but
20 might bind in the future. In this sense the optimal policy is "prudential" or "precautionary"
21 in nature. Intuitively, since τ_t^B is impotent during the crisis, the best thing that policy can

do, conditional on having only the tax on debt as instrument, is to reduce the probability that a crisis occurs by limiting the amount that agents borrow in equilibrium (i.e. by taxing B_{t+1}). We also note here that, in the region in which the constraint binds ($\lambda_t > 0$), any value of τ_t^B can implement the social planner allocation.[18]

Figure 2 plots the policy function of τ_t^B, for a negative one-standard deviation shock, that solves the optimal policy problem above and replicates the SP allocation, as well as the welfare gains for τ_t^B as a function of current bond holdings. Figure 3 reports the ergodic distributions of debt in the CE and the SP allocations. Table 1 reports the ergodic mean of debt as a share of (annual) income in units of tradable consumption, the unconditional probability of a financial crisis in the model, as well as the average welfare gain associated with this policy instrument relative to the CE.[19]

Intuitively, when the economy approaches the binding region the tax rate goes to zero; before the crisis hits, the higher is the probability that the constraint binds, the higher is the tax on borrowing. Looking at the welfare gains we can see that they also peak when the constraint binds, but revert to zero slower than the tax rate. The welfare gains of optimal capital controls persist past the level of debt at which the constraint binds because entering a crisis with less debt makes the crisis relatively less costly (see Figure 8 below and its discussion on this latter point). As we can see from Figure 3 and Table 1, the policy intervention reduces the debt/income ratio and the likelihood of a financial crisis. This implies that the economy, on average, will borrow less under the optimal capital control policy than in the competitive equilibrium and will experience fewer and less costly financial crises.

4 Optimal exchange rate policy

We now consider the use of consumption taxes or equivalently real exchange rate interventions. We first examine the nontradable consumption tax. As we shall see, the tax on tradable goods achieves the same results when used optimally.

4.1 Nontradable tax

Like before, let us first define the Ramsey problem when τ_t^N is the policy instrument.

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[18]This is true as long as τ^B is less than $\bar{\tau}^B$, i.e. the maximum value of the tax rate consistent with the constraint being binding.

[19]See the appendix on the the solution method for the SP allocation and the computation of the welfare gains.

Definition 2 *For a given $\{B_0\}$ and assuming that $\{Y_t^T\}$ and $\{Y_t^N\}$ are Markov processes with finite, strictly positive support, the Ramsey problem for τ_t^N as instrument is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t u\left(C_j\right) \right\},$$

subject to the resource constraints (7) and (8), the government budget constraint (13), the borrowing constraint

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N \right]. \tag{20}$$

and the first order conditions of the household

$$u'(C_t)C_{C^T} = \lambda_t + \beta \left(1 + r\right) E_t \left[u'(C_{t+1})C_{C^T} \right], \tag{21}$$

$$\frac{(1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}}} = P_t^N(1 + \tau_t^N). \tag{22}$$

It is noteworthy that the non-tradable consumption tax directly affects the relative price of nontradables (i.e. the real exchange rate). Note also that, in normal times and in the unconstrained equilibrium, the determination of the consumption of tradable and non-tradable goods is independent from P_t^N. Therefore, τ_t^N is neutral when the constraint does not bind. In fact, the Euler equation and the goods market equilibrium conditions are all that is needed to determine consumption of tradables and nontradables when the constraint does not bind. In contrast, when the constraint binds, τ_t^N is no longer neutral because changes in P_t^N affect the value of the collateral, and hence the consumption of tradable goods.

The next proposition says that, when used optimally, this consumption tax can achieve the unconstrained allocation (i.e., it assures that the borrowing constraint is never strictly binding in the equilibrium of our economy so that $\lambda_t = 0$ for all t). To characterize the solution of this Ramsey problem we follow the same two steps of the previous proposition. First we characterize a policy rule for τ_t^N that decentralizes the unconstrained competitive equilibrium. Then we show that this equilibrium is the one that solves the Ramsey problem above.

Proposition 2 *In an economy defined by (1), (4), (12) and (13) with a tax on non-tradable consumption τ_t^N as the government instrument, there exists a policy for τ_t^N that decentralizes the unconstrained allocation. This policy is Ramsey optimal and time-consistent.*

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₂₂   PROOF: see online appendix.

₁   Several remarks are also in order here. The proposition above implies that real exchange
₂ rate policy always dominates capital control policy in welfare terms. Under this policy, it is
₃ possible to undo the constraint completely and replicate the unconstrained equilibrium. In
₄ contrast, capital controls can only limit (by reducing the probability of hitting the constraint)
₅ the distortionary effects of the pecuniary externality associated with the constraint, but not
₆ the constraint itself. As we can see from Table 1, these welfare differences are quantitatively
₇ very large.

₈   How does this policy work? The intuition for the result is that (22) directly links the tax
₉ rate to the relative price of nontradables. When the borrowing constraint does not bind, the
₁₀ policy tool is neutral in the sense that it affects P_t^N, but not the consumption allocation. In
₁₁ contrast, when the constraint binds, the tax is no longer neutral and can be used to affect
₁₂ the value of collateral in the borrowing constraint, and hence also tradable consumption.
₁₃ By subsidizing the consumption of nontradable goods, the policy increases its relative price.
₁₄ Crucially, when the constraint binds, a higher relative price increases the value of collateral
₁₅ and avoids the debt-deflation mechanism that would otherwise ensue.[20]

₁₆   In equilibrium, agents anticipate that this policy will undo the constraint when it binds
₁₇ and will behave as if the constraint does not exist (i.e. like in the unconstrained allocation).
₁₈ As we can see from Figure 1, for a given endowment of nontradable goods, the uncon-
₁₉ strained allocation (UE) entails a much higher price of nontradables and consumption of
₂₀ tradable goods during tranquil times than in the two constrained allocations (the CE and
₂₁ SP). Eventually (i.e. in finite time) our economy will hit the borrowing constraint because
₂₂ agents are relatively impatient. When that happens, under the optimal policy, τ^N will be
₂₃ set so that the multiplier on the constraint is zero (i.e. the constraint is just binding).

₂₄   Notice that the policy function for τ^N is time-consistent, and hence promising to elimi-
₂₅ nate the borrowing constraint by supporting the relative price of nontradable whenever the
₂₆ constraint binds is fully credible in equilibrium.

₂₇   This optimal policy can also be implemented with a fixed tax rate. Since any policy
₂₈ schedule $\tau_t^N \in (-1, \hat{\tau}_t^N]$ can achieve the unconstrained allocation, for any $\hat{\tau}_t^N$ that undoes
₂₉ the borrowing constraint, there also exists a fixed subsidy $\tau_{fix}^N \in (-1, \hat{\tau}_t^N]$ that replicates such
₃₀ allocation. This consideration is important because it simplifies the practical implementation
₃₁ of the policy. Indeed, for a given exogenous state $\{Y_t^T, Y_t^N\}$ with finite support, it is possible

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[20]For this reason, in broader terms, we can interpret this policy as a collateral support policy. In this specific case of our model it takes the form of a fiscal policy intervention aimed at supporting the relative price of non tradeable that enters the borrowing constraint.

to determine the corresponding fixed subsidy for which the constraint does not bind. The fixed level of τ^N_{fix} will be such that $\tau^N_{fix} \in (-1, \hat{\tau}^N_{t,\min}]$, where $\hat{\tau}^N_{t,\min}$ is the lowest value of the subsidy given the finite support of the exogenous states.

Welfare gains from optimal this policy tool are two orders of magnitude higher than the gains from implementing the SP allocations (Table 1). Figure 4 plots the implied optimal τ^N as a function of current bond holdings and the associated welfare gains for a negative one-standard deviation shock. The implied subsidy and the welfare gains associated with it increase with the level of existing debt. As we can see from Figure 4, this optimal policy subsidizes nontradable consumption, limiting the downward pressure on the relative price of non tradable goods. As a result, agents can borrow and consume much more in both good and bad times. In this case, however, the probability of a crisis is zero, despite the fact that borrowing and consumption are much higher than in the CE or the SP (Table 1).

We note here that, for our calibration (which is the same as in Bianchi, 2011), agents are very impatient and the incentive to borrow dominates the precautionary motive that tends to contain their borrowing. The relative strength of this "impatience" effect implies that even when the initial net foreign assets position is positive, agents will borrow up to the borrowing limit, so that a tax subsidy on nontradable consumption is needed to relax the credit constraint. As the current debt position worsens, the state contingent tax subsidy becomes bigger, tending towards the lower bound of -1.

To quantify what a more realistic policy can achieve in welfare terms, we consider the case of a fix, 10 percent non tradable subsidy. Such a policy yields an average relative price of on nontradables that is approximately 10 percent less depreciated than in the competitive equilibrium with an average welfare gain of 0.4 percent of permanent consumption. This is about the same as that attained with the optimal capital control policy, which nonetheless is a state contingent tax schedule (Table 1).

4.2   Tradable tax

We now discuss the last policy tool available (τ^T_t), leaving the details in the online appendix. The tax on tradable consumption affects not only the intratemporal relative price, but also the intertemporal allocation of resources through the Euler equation. Despite this difference, we show in appendix that it is possible to find a policy for τ^T_t that replicates the unconstrained allocation, like in the case of the nontradable consumption tax τ^N_t. The difference between the two policies is that the subsidy on nontradable consumption requires financing through lump sum taxes, while the revenues from the tax on tradables will be re-

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bated as lump sum transfers to private agents. From a practical standpoint, this is important as fiscal space is typically limited in the midst of a financial crisis.

Both policy tools (τ_t^T and τ_t^N) could be interpreted strictly in terms of fiscal policy actions or more broadly as policy aimed at targeting the real exchange rate. In a way the latter interpretation is related to the recent literature that proposes to manipulate the real exchange rate through the use of fiscal tools—see for instance Lipinska and Von Tadden (2009), Franco (2011) and Fahri, Gopinath and Itskhoki (2013). The difference here is that we want to limit the depreciation of the real exchange rate for financial stability purposes, while in the literature on fiscal devaluation the idea is to engineer a devaluation to gain competitiveness.

5   Optimal capital controls and exchange rate policy with distortionary financing

Our analysis in the previous section showed that exchange rate policy dominates capital control policy in welfare terms. Intuitively, in a debt-deflation environment, optimal policy aims at relaxing the collateral constraint. In particular, we have shown that optimal policy (in the form of taxes on consumption) supports the relative price that influences the borrowing constraint and in principle can undo the effects of a binding constraint completely. The result hinges on the ability of the Ramsey planner to manipulate the price of the collateral without costs, because our policy instruments operates in the context of a balanced government budget in which lump sum transfers or taxes are available.

We now depart from this key assumption by considering an environment in which lump-sum transfers/taxes are not available[21], so that it is costly to manipulate the price of the collateral, which is the real exchange rate in our model. Such cost can be interpreted more broadly as representing another distortion in the economy, a second objective of exchange rate policy, or any situation in which managing the real exchange rate during a financial crisis is difficult.[22]

Given the structure of our endowment economy, we consider two possibilities for the government budget constraint. In the first one, the set of taxes is arbitrarily restricted to τ_t^B and τ_t^N. In the second one, we allow the use of all the tax instruments discussed thus far,

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[21]In a model with heterogenous agents, lump sum instruments will have distributional implications. Here we abstract from these issues but still we consider an environment in which lump sum instruments are not available.

[22]We discuss further this interpretation in the next section of the paper.

τ_t^B, τ_t^N, and τ_t^T.

5.1 Two policy instruments

In the first case, the government budget constraint becomes:

$$\tau_t^B B_{t+1} = \tau_t^N P_t^N C_t^N, \tag{23}$$

and the government budget is balanced by combining the tax on borrowing with a subsidy on nontradable goods. The following definition states the corresponding Ramsey problem.

Definition 3 *For a given $\{B_0\}$ and assuming that $\{Y_t^T\}$ and $\{Y_t^N\}$ are Markov processes with finite, strictly positive support, the Ramsey problem for τ_t^N and τ_t^B as instruments, when (23) holds is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t u \left(C_t^j \right) \right\},$$

subject to (7) and (8) and (20), and the first order conditions of the households

$$u'(C_t)C_{C^T}(1 + \tau_t^B) = \lambda_t + \beta (1 + r) E_t \left[u'(C_{t+1})C_{C^T} \right], \tag{24}$$

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{-\frac{1}{\kappa}}} = P_t^N(1 + \tau_t^N). \tag{25}$$

As we cannot characterize the solution of this problem analytically, we must rely on numerical methods. To do so, we note first that, given our chosen instruments (i.e. τ_t^N and τ_t^B), the problem is time consistent.[23] We then use a computational method that exploits the Markov-Perfect nature of the equilibrium, proposed by Benigno et al. (2012) and summarized in the appendix. For comparison purposes, the economy continues to be calibrated exactly as in Bianchi (2011). Here we report and discuss only the solution.

Figure 5 plots the policy function under the optimal policy for τ_t^N and τ_t^B and the associated welfare gains in terms of tradeable consumption as a function of current bond holdings for a negative one-standard deviation shock. Figures 6 describes the policy function for

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[23]To see this, note that we can reduce the optimal control problem to a time-consistent static problem by considering the restricted problem in which the Ramsey planner maximizes agents' utility subject to (7), (8), (20) and (25). We can then solve for the allocations, the multiplier on the credit constraint and the relative price. Next, we use (23) and (24) to retrieve the path of taxes. In the appendix we provide an alternative proof based on the equivalence between the commitment and the time-consistent problem.

B_{t+1}, C_t^T and P_t^N under the optimal policy (OP, dashed line) and the competitive allocation (CE, solid line). Figure 7 reports the ergodic distribution of debt. In order to assess the severity of the crisis, Figure 8 also reports the ergodic distribution of total consumption growth in unit of tradable consumption during crisis times (i.e., the change in consumption from $t-1$ to t, given that the economy is in a financial crisis in period t). For this purpose, a crisis is identified, as in Bianchi (2011), by a constraint that binds strictly and a debt reduction larger than one-standard deviation. In these plots, the constraint binds at a level of debt of about -0.95, where the policy rules display a kink.

As we can see from Figure 5, when exchange rate policy is costly, there is scope for both ex-ante and ex-post interventions. During normal times, the optimal policy requires capital controls whose revenues are rebated in the form of subsidies to nontradable consumption; during crises times, the optimal policy requires subsidies to non-tradable consumption to limit the depreciation of the relative price of nontradable goods, financed by a tax on the amount that agents borrow.

The optimal policy depends crucially on the interaction between ex-ante and ex-post interventions. In the context of our simple economy, this interaction is affected by the way the policy interventions are financed. When financing of ex-post intervention is not costly (i.e. there are lump-sum taxes) policies aimed at supporting the market price of collateral are fully effective and can achieve the unconstrained allocation. In contrast, when financing of ex-post intervention is distortive, preventing excessive depreciation of the real exchange rate becomes costly, and the optimal policy weights the marginal benefit of relaxing the borrowing constraint with the distortion introduced by capital controls. Indeed, when the constraint binds, the tax on debt affects C_{t+1} through (24). Since the ex-post policy becomes costly, it is no longer fully effective in addressing the pecuniary externality, and it becomes optimal to intervene during normal times to reduce the probability of meeting the borrowing constraint. Consistent with this, we can see from a comparison of Figure 2 and Figure 5 that the optimal capital control tax, in the region where the constraint is not binding, is much smaller than the case in which capital control is the only government instrument.

There are three other features of the optimal policy that are noteworthy. First, we note that when the constraint is not binding, while the tax on the amount that agents borrow affects their borrowing decision, the subsidy to nontradable consumption is neutral and is equivalent to a lump-sum transfer. On the other hand when the constraint binds, both instruments affect the real allocation.

Second, in this set up, there continues to be more borrowing and consumption than in the competitive equilibrium despite the fact that the economy experiences fewer and less severe

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crises (see Table 1 and Figure 8). The Ramsey planner achieves this by choosing a different allocation of consumption, with relatively more consumption of tradable goods compared to the competitive equilibrium allocation. As a consequence, the welfare gains of this optimal policy mix are more than twice as large as those in which only capital controls are used, and continue to be larger the more indebted is the economy (see Figure 5 and Table 1). This is consistent with the old adage that where borrowing is allocated is at least as important as how much borrowing takes place.

Third and finally, agents borrow more than in the competitive equilibrium allocation during normal times even though optimal policy requires a tax on the amount agents borrow (Figure 6 and 7). Intuitively, on the one hand, agents want to borrow less because their borrowing is taxed; on the other hand, they are willing to borrow more since crises events are mitigated (only in part in this case) by policy intervention (see Figure 8 and Table 1, respectively). Indeed, the real exchange rate depreciates less during crises times compared to the competitive equilibrium allocation and allows agents to consume more (Figure 6). In this setting, therefore, the rationale for capital controls is not related to the amount that agents borrow in the unregulated economy, the so called "overborrowing" on which the existing literature focused on, but rather to the relative (in)effectiveness of the ex post intervention.

5.2   Three policy instruments

Consider now a second possibility in which all available distortionary taxes can be combined to balance the budget:

$$\tau_t^B B_{t+1} = \tau_t^N P_t^N C_t^N + \tau_t^T C_t^T. \tag{26}$$

In this situation, it is possible to show that there is a combination of policy tools that can achieve the unconstrained allocation even if there are no lump sum transfers/taxes. In the appendix, we prove that we can always combine the triplet of policy tools $(\tau_t^N, \tau_t^T, \tau_t^B)$ to undo the international borrowing constraint.

The policy implication of this last exercise echoes what we emphasized earlier: the set of instruments and their relative effectiveness is crucial for the optimal policy design. The third instrument addresses the distortion introduced by the second one in crisis times. Intuitively, it is possible to use the tax on tradable goods to undo the efficiency losses caused by the use of tax on borrowing when policy aims at supporting the real exchange rate. This is consistent with the notion that with enough instruments we can always undo a friction. In our context, this implies that the challenge for the policy maker is to identify the specific combination of instruments that are most effective in addressing the pecuniary externality

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and its interaction with the others relevant frictions in the economy.

5.3   Extensions

Our analysis shows that, in economies with occasionally binding collateral constraints, it is optimal to design policies aimed at relaxing the constraint when it binds, and the desirability of policies aimed at preventing crisis depends on the cost-effectiveness of such crisis resolution interventions. Here we want to discuss some extensions of the basic framework and show how the general message of our optimal policy analysis is a robust feature of this model environment.

Imperfect exchange rate intervention   We first consider a more realistic case of costly (or less than fully effective) ex-post intervention, which we label "imperfect exchange rate intervention," for example, because it is not be feasible to perfectly control the real exchange rate during a crisis event. In practice, this might be due to imperfect credibility or because of limited availability of foreign exchange reserves needed to support the nominal exchange rate, as we discuss in section 6 below.

To model this idea (fully developed in the appendix), we assume that when it is not feasible to implement the desired level of the subsidy τ^N, τ^N must be set to zero. In appendix, we show that under this policy it is not always possible to relax the collateral constraint, so that it becomes optimal to intervene ex-ante, from a Ramsey planner perspective, like in the case of distortionary financing discussed above. Intuitively, when exchange rate intervention is imperfect, crises become more costly events, and it is desirable to tax borrowing in normal times to limit the probability that a crisis occurs.

Borrowing constraint with post-tax income   The second extension that we consider is the situation in which the borrowing constraint depends on post-tax income as follows:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N - T_t\right].\tag{27}$$

It is evident that, under this assumption, changes in τ^N financed through T_t, will not relax the borrowing constraint. Despite this, it is possible to show that the general principle of optimal policy design in this class of models that we stressed above continues to hold, but it may require to use a different policy tools. In fact, in this specific case, the policy maker needs use the tax on borrowing, τ^B, along with lump-sum transfers/taxes to relax the constraint. When (27) holds and binds, the Ramsey planner will increase the value

of the collateral by transferring resources to the household via T_t, at the a cost of higher τ^B. Indeed, in the appendix, we show that this case is isomorphic to the case in which the borrowing constraint depends on pre-tax income, and the available policy tools are τ^N and τ^B. The only difference is that, in this case, the borrowing constraint is relaxed through T_t rather than by engineering an increase P_t^N.

Production Lastly, we note that, when we have a production economy, and hence multiple margins on which the pecuniary externality can distort decisions—see for instance the economy analyzed by Benigno et al. (2012, 2013)—it is possible to show that exchange rate policy alone via τ^N cannot restore constrained efficiency or remove the borrowing constraint. For example, the optimal policy for τ^N alone, in that more general setting, is a tax in normal times and a subsidy in crisis times. That policy therefore has both a crisis prevention element aimed at containing the frequency of financial crises, and a crisis resolution element aimed at mitigating their effect by relaxing the constraint when they do occur. To restore constrained efficiency or remove the borrowing constraint altogether, however, would require the use of multiple tools. Indeed, in that model environment, collateral price support policies induce distortions in the allocation of labor between tradeable and non-tradeable production that requires the use of an offsetting policy tool to fully restore efficiency or contain the cost of trying to remove the constraint.

6   Discussion

Our model is useful to discuss exchange rate policy in the real world, and the implied "optimal" policies are consistent with the experience of emerging market economies over the past 20 years or so.

   A first implication of our analysis is on the role of collateral price support policies when they can be implemented in a costless way. In the context of our model, these policies take the form of a fiscal subsidy to the consumption of nontradable goods, financed in a lump sum manner. This contains the fall of the relative price of nontradable (or the depreciation of the real exchange rate in our model) that typically occurs during a sudden stop of capital inflows. If such a policy is feasible, our analysis shows that, not only it contains the crisis when one occurs, but it also eliminates the scope for any prudential measure such as capital controls. This is because the intervention can removes the borrowing constraint altogether, which is the only source of inefficiency in our model economy. Of course, in reality, there are other distortions, possibly leading to different conclusions and we discuss this possibility

below.

Which polices in the real world can support the real exchange rate and how costly are they to implement? In practice, the real exchange rate is typically supported by defending the nominal exchange rate by selling previously accumulated foreign exchange reserves. And while accumulating or borrowing foreign exchange reserves is costly, drawing them down at any particular point in time is costless.

For example, during the global financial crisis, Brazil and Mexico faced a sudden stop in private capital inflows following the Lehmann's collapse in September 2008. The Brazilian Real depreciated by more than 20 percent in a month against the US dollar, and the central bank intervened heavily to defend it as predicted by our model. As Mesquita and Toros (2010) emphasize, the main motivation for this intervention was the vulnerability of the non-financial corporate sector to the depreciation of the Real because of their exposure in the derivative market to US dollar swaps (proxied in our model with borrowing in units of tradable consumption). A similar experience was shared by Mexico when large corporate entities were also exposed to foreign currency derivatives at the time of Lehmann's collapse. In their account of the Mexican experience, Chang, Cespedes and Velasco (2012) emphasize how the response of the policy authorities consisted in foreign exchange market intervention with the objective to limit the depreciation of the Mexican Peso in the face of currency mismatches in the corporate sector balance sheet.

More broadly, in the context of the recent US and European financial crises, the prescription of our model can be interpreted as interventions that avoid the collapse of asset prices when a crisis occurs. In this sense, our results not only rationalizes the need to set a floor under the exchange rate as in the emerging market crises of the 1990s and the 2000s, but also the non-conventional policies of purchasing risky assets to contain "fire sales" and the asset deflation spirals that characterized the United States and European crises.[24]

Official reserves however are always limited, and this limited availability exposes countries to costly speculative attacks. Many emerging market countries learned this lesson the hard way in the 1990s and the 2000s as speculative attacks on limited pools of foreign exchange reserves broke many pegs: Mexico in 1994, Thailand in 1997, South Korea and Indonesia in 1998, Russia in 1998, Brazil in 1999, Argentina in 2001, Uruguay in 2002, etc. Once out of reserves, these countries had to borrow foreign currency from the IMF under tight macroeconomic adjustment programs to contain the initial devaluations. Indeed, as predicted

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[24]It is possible to show that the small open economy studied here is isomorphic to an environment in which domestic banks intermediate foreign saving and households borrow using a domestic asset as collateral. In that context, optimal ex post policy, when warranted, supports domestic asset prices.

by our model, supporting the exchange rate was a crucial component of all adjustment programs supported by the IMF in Indonesia, South Korea, and Brazil during the period 1997-1999 (IMF Independent Evaluation Office, 2003). These adjustment programs turned out to be economically and politically very costly. As a result, after these crises, countries started to accumulate very large pools of official reserves to deploy in support of the exchange rate in the case of sudden halt in capital inflows as they did during the global financial crisis.[25] But even when accomplished gradually rather than borrowing from the IMF, reserve accumulation is costly. Countries must save in a precautionary manner in a riskless asset while continuing to borrow in risky instruments as long as they are in a net debtor position. So there is a carry cost, or premium for holding reserves.

Perhaps even more importantly, exchange rate policy also has other objectives than that of maintaining financial stability by mitigating the effects of currency mismatches. Exchange rate policy is typically tasked to also address competitiveness issues and to contribute to macroeconomic stability by helping to manage inflation. In our model, financial stability is the only policy objective, as there are no other frictions justifying intervening in the economy for macroeconomic stabilization or competitiveness reasons.

If we were to introduce other frictions and hence policy objectives in our model, a trade off would emerge similar to the one we studied in the previous section by assuming distortionary financing of exchange rate policy. One example is a government that targets the real exchange rate by manipulating the nominal parity in the presence of both a borrowing constraint like ours and a nominal rigidity. In this case, the advantage of keeping the exchange rate relatively appreciated is to support the agents' borrowing capacity. The disadvantage of would be to cause unemployment in response to shocks. Our distortionary cost captures the essence of costly ex post interventions in the presence of other distortions or government objectives.[26]

Indeed, the second main policy implication of our analysis is that, if financial crises cannot be contained or mitigated without incurring significant costs, or there are additional distortions to consider, a policy of crisis prevention becomes part of the optimal policy mix, such as for instance using capital controls in a countercyclical manner. However, when both ex ante and ex post interventions are used jointly under the optimal policy mix, the level

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[25]Emerging markets official reserves (excluding gold) increased from about one trillion US dollar in 2000 to over 6 trillions in 2012 according to IMF IFS data (or about a third of world GDP valued at current US dollars). While this spectacular accumulation of reserve assets cannot be explained entirely by prudential or precautionary motives, most empirical studies concur that precautionary saving is the most important determinant of this process.

[26]Ottonello (2015) studies optimal exchange rate policy in such a set up and illustrates the trade off between increasing agents' borrowing capacity (dubbed credit access policy) and unemployment. His analysis, however, considers a restricted sets of policy instruments.

of the tax on borrowing is much lower than when the capital control is used as the only instrument (cfr. Figure 2 and 5).

This latter result is consistent with available empirical evidence on the use of capital controls. As Fernandez, Rebucci, and Uribe (2015) pointed out, if countries were to use capital controls in a prudential manner as implied by Proposition 1, we should observe active use of countercyclical capital controls. However, when Fernandez et al (2015) looked at a large number of countries over the period 1995-2011, found that capital controls are virtually flat during episodes of boom and bust in output or the current account.

In summary, we conclude from this review of country experiences over the past 20 years or so that, while the mechanics of optimal policies implied by our model are different than those typically implemented in the real world, the general principle followed by these policies is very much consistent with them. The policies implied by our analysis are also consistent with the available empirical evidence on the use of capital controls, in stark contrast to those implied by the existing literature.

7   Conclusion

In response to the recent global financial crisis, a new policy paradigm emerged in which old fashioned forms of government interventions such as capital controls and other quantitative restrictions on credit flows—the so called macro-prudential policies—have become part of the standard policy toolkit. Arguably macro-prudential policies are desirable because they can help prevent financial crises that otherwise would be too costly to endure or contain with only ex post interventions.

In this paper we study the optimal policy mix of ex post, crisis management policy tools and ex ante, crisis prevention policy tools. We first show that when the Ramsey planner can choose among different policy tools, ex post collateral price support policies dominate prudential policy measures in welfare terms by two orders of magnitude. This dominance is conditional on the extent to which price support policies do not entail efficiency losses. Indeed, when collateral price support policies can be used effectively, there is no need for macro prudential policies. In contrast, when crisis management policies are not fully effective because they are costly to implement, ex-ante policies such as capital controls can be rationalized as a complement to collateral price support policies that limit the occurrence of crises. The joint use of ex ante and ex post policies achieves a welfare gain of more than 1 percent of permanent consumption in our model; a gain that is twice as large as the welfare gain of using only capital controls.

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Our analysis is conducted in the context of a relatively simple quantitative model, but in reality the trade-offs that policymakers face are richer that the ones implied by our framework. For instance, there are benefits from a more depreciated exchange rate in terms of the classical expenditure switching effect of exchange rates that are not incorporated into our analysis. To an extent, we can interpret our model as one in which balance-sheet consideration dominates other exchange rate policy motives, but we acknowledge that a richer model would be needed to quantify these issues. We regard the study of optimal monetary and macro-prudential policy in a quantitative model in which pecuniary externalities interact with nominal rigidities as an area of fruitful future research.

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Table 1: Ergodic Averages

|	Debt to Income	Prob. of Crisis	Welfare Gain
CE	-29.2%	6.7%	NA
SP	-28.4%	1.2%	0.41%
UE	NA	0.0%	33.8%
OP	-30.5%	4.9%	1.10%

Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; UE the unconstrained equilibrium; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The table reports ergodic means (in percent). Welfare gains are relative to the CE and are measured in unit of tradable consumption.

Figure 1: Alternative Allocations: Decision Rules
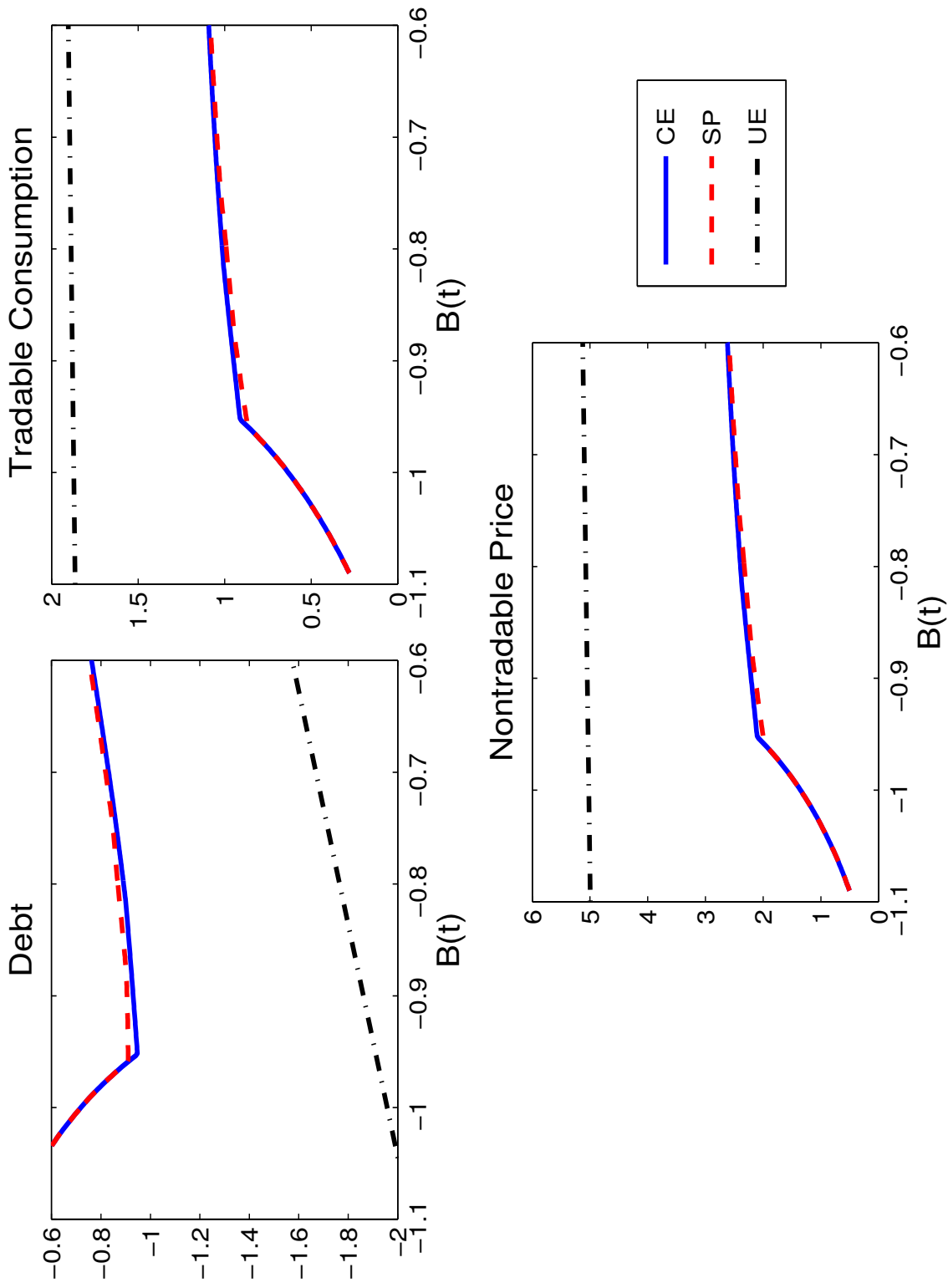
Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; UE the unconstrained equilibrium. The figure plots the equilibrium decision rules or policy functions of the endogenous variables plotted conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 2: Optimal Capital Control Policy
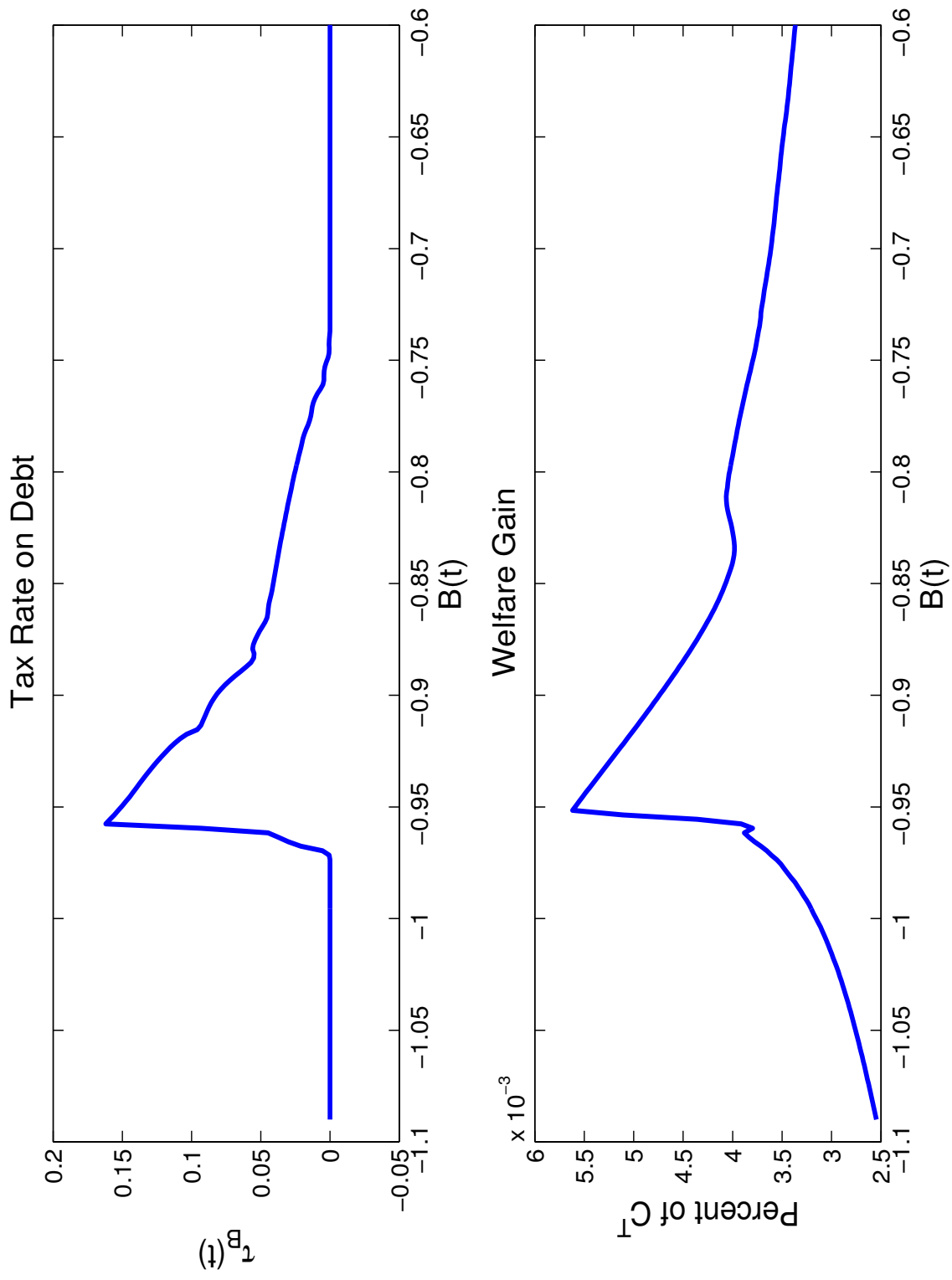
Notes: The figure plots the optimal debt tax rate and the associated welfare gain relative to the competitive equilibrium conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 3: Optimal Capital Control Policy
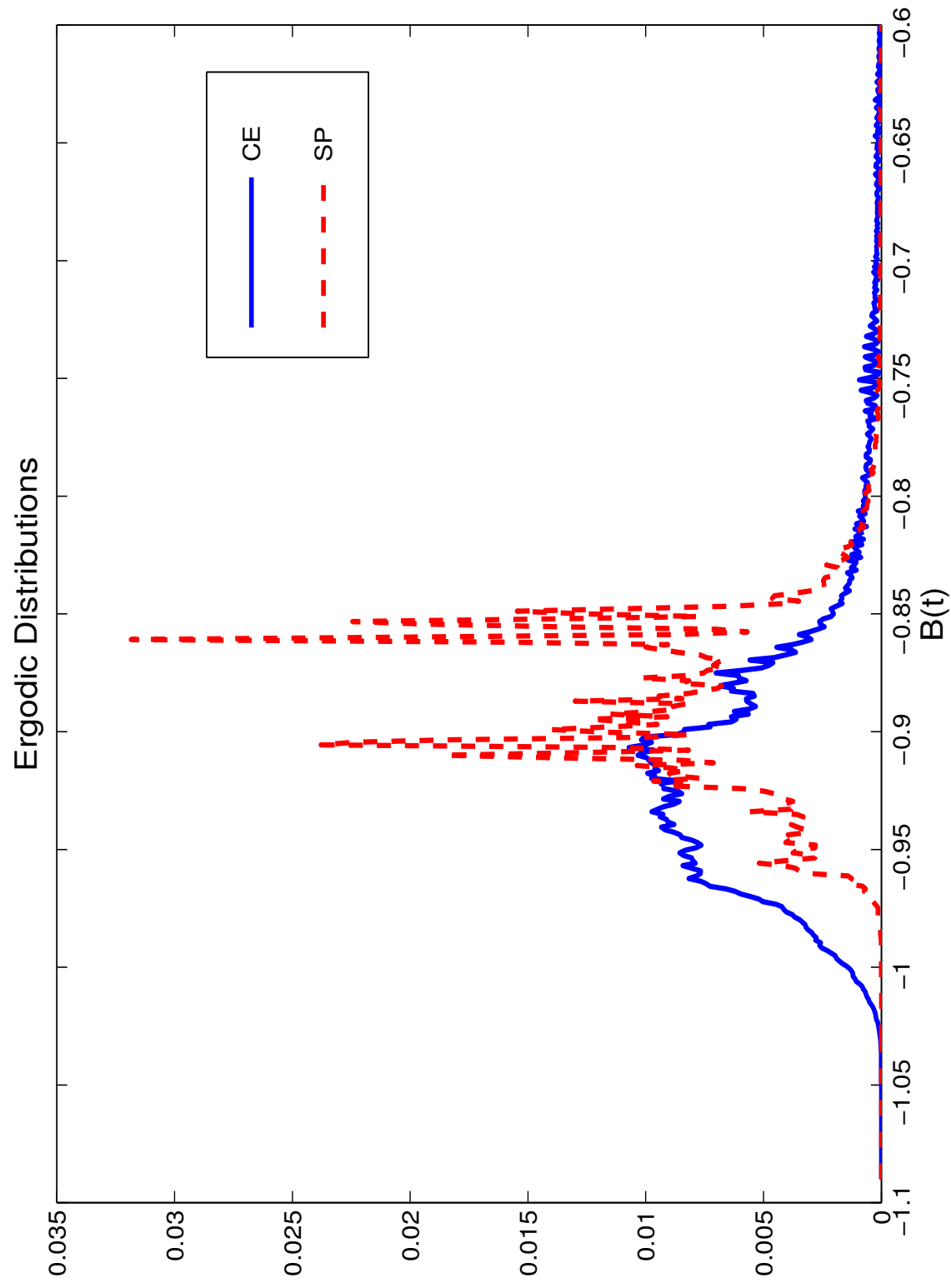
Ergodic Distributions

Notes: The figure plots the ergodic distribution of debt in units of tradable consumption in the competitive equilibrium (CE) and the social planner allocations (SP). Borrowing decreases from left to right on the x-axis.

Figure 4: Optimal Exchange Rate Policy

Notes:The figure plots the optimal nontradable consumption tax rate and the associated welfare gain relative to the com-petitive equilibium conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 5: Optimal Capital Control and Exchange Rate Policy

Notes: The figure plots the optimal debt and nontradable consumption tax rates and the associated welfare gain relative to the competitive equilibrium, conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.
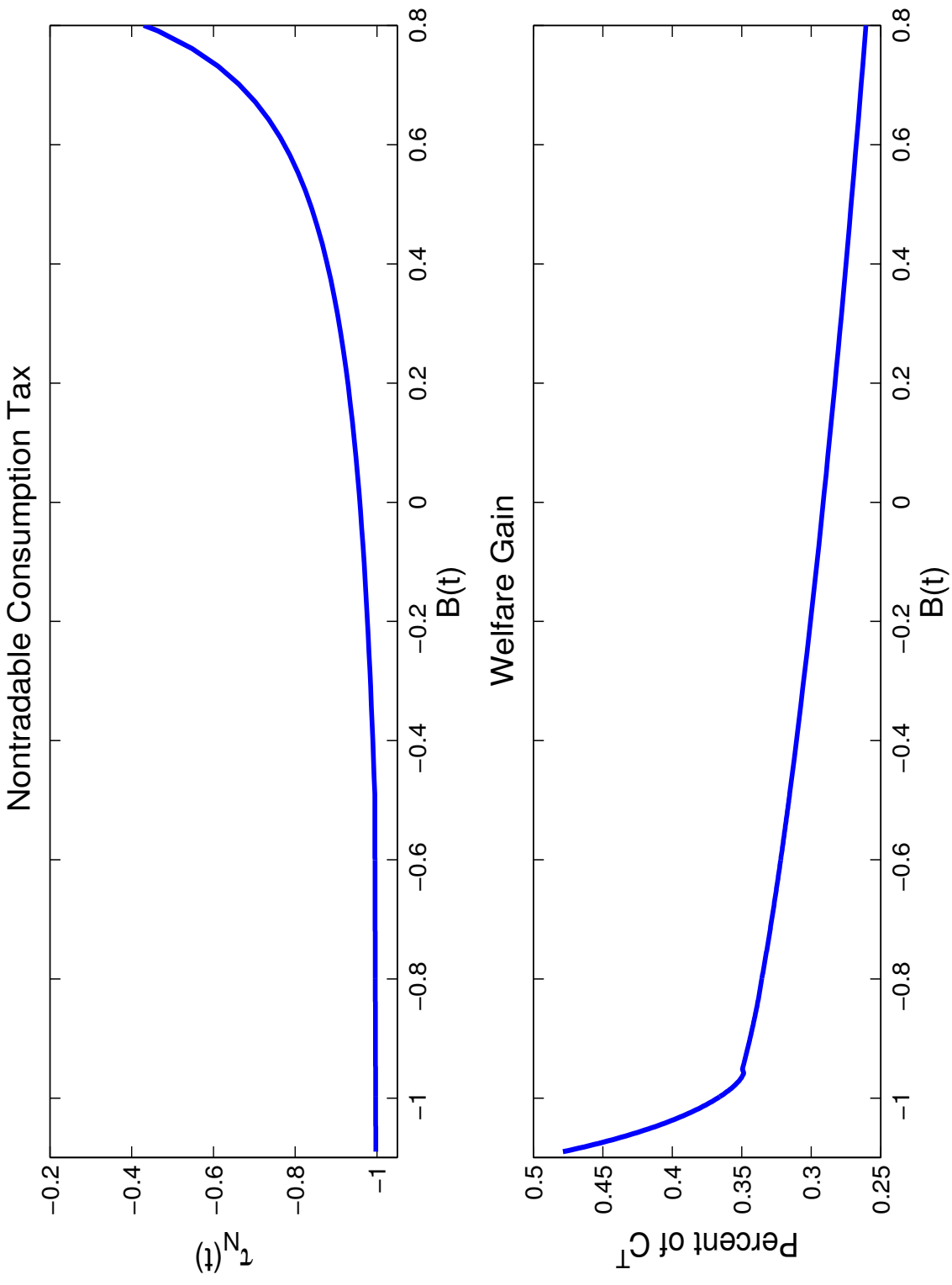
Figure 6: Optimal Capital Control and Exchange Rate Policy: Decision Rules
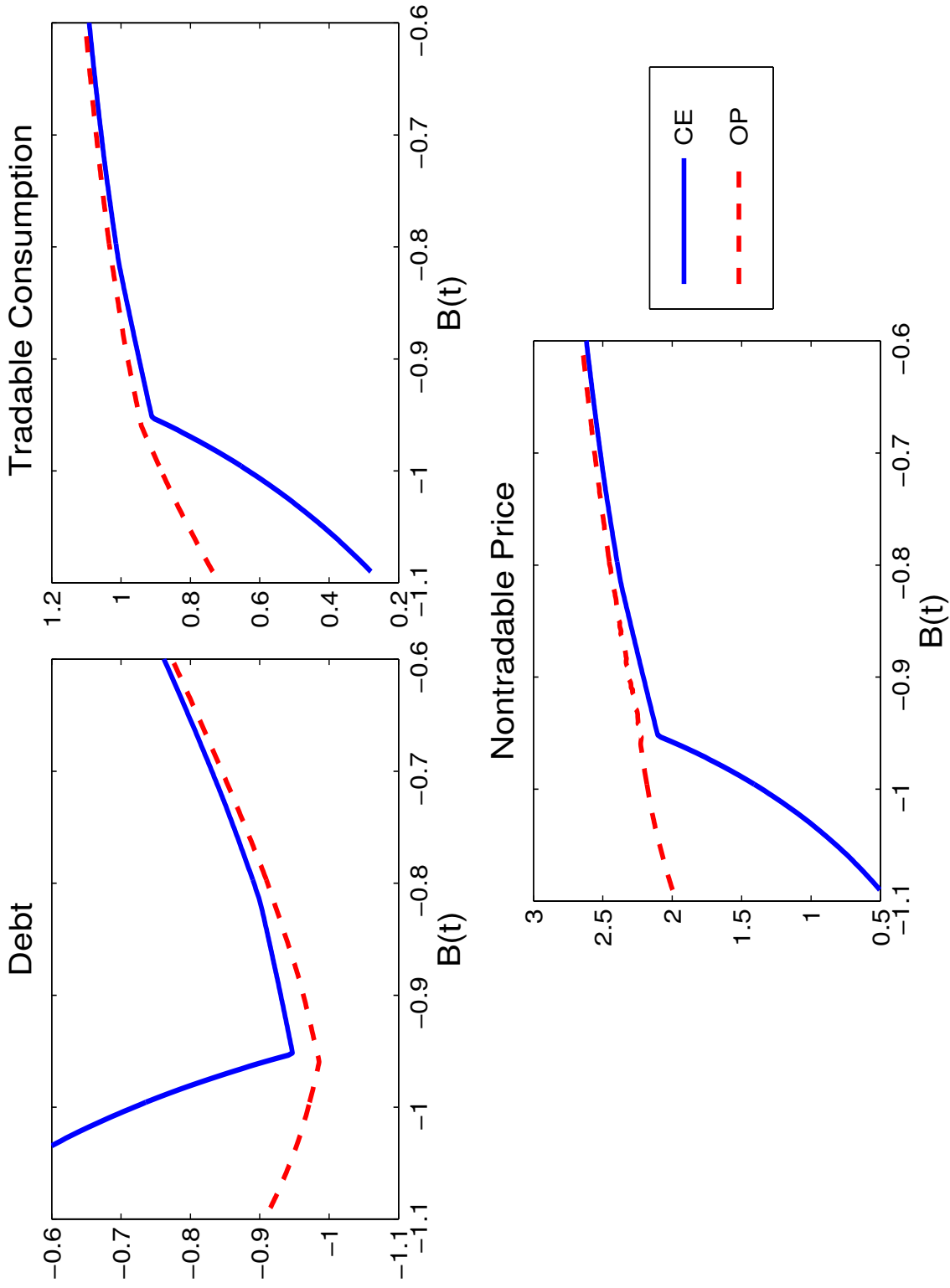
Notes: CE denotes the competitive equilibrium allocation; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The figure plots the equilibrium decision rules of the endogenous variables plotted conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.
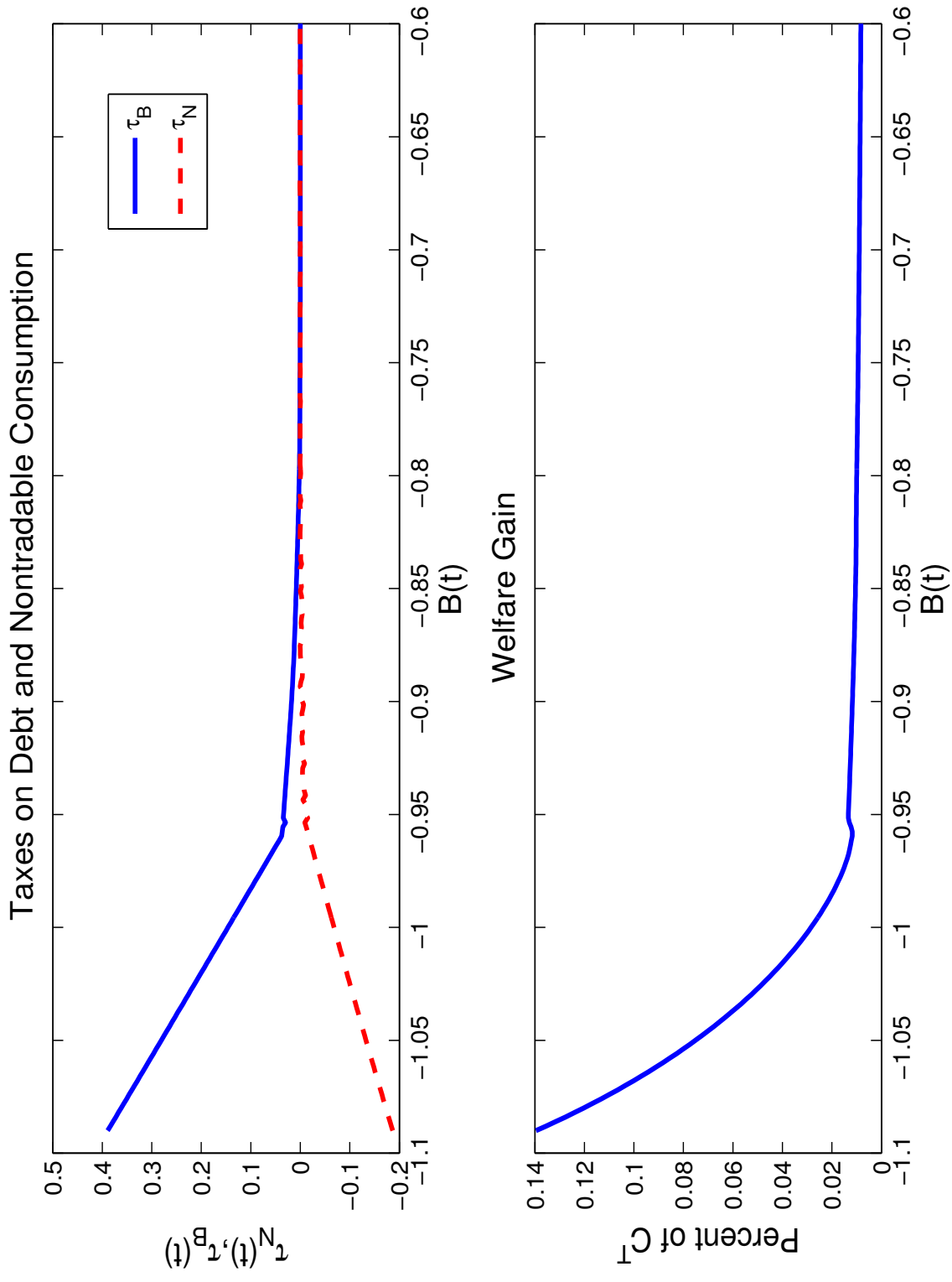
Figure 7: Optimal Capital Control and Exchange Rate Policy

Notes: The figure plots the ergodic distribution of debt in units of tradable consumption in the competitive equilibium (CE) and the optimal policy equilibrium with both debt tax and nontradable consumption tax (OP). Borrowing decreases from left to right on the x-axis.

Figure 8: Comparing Policy Regimes
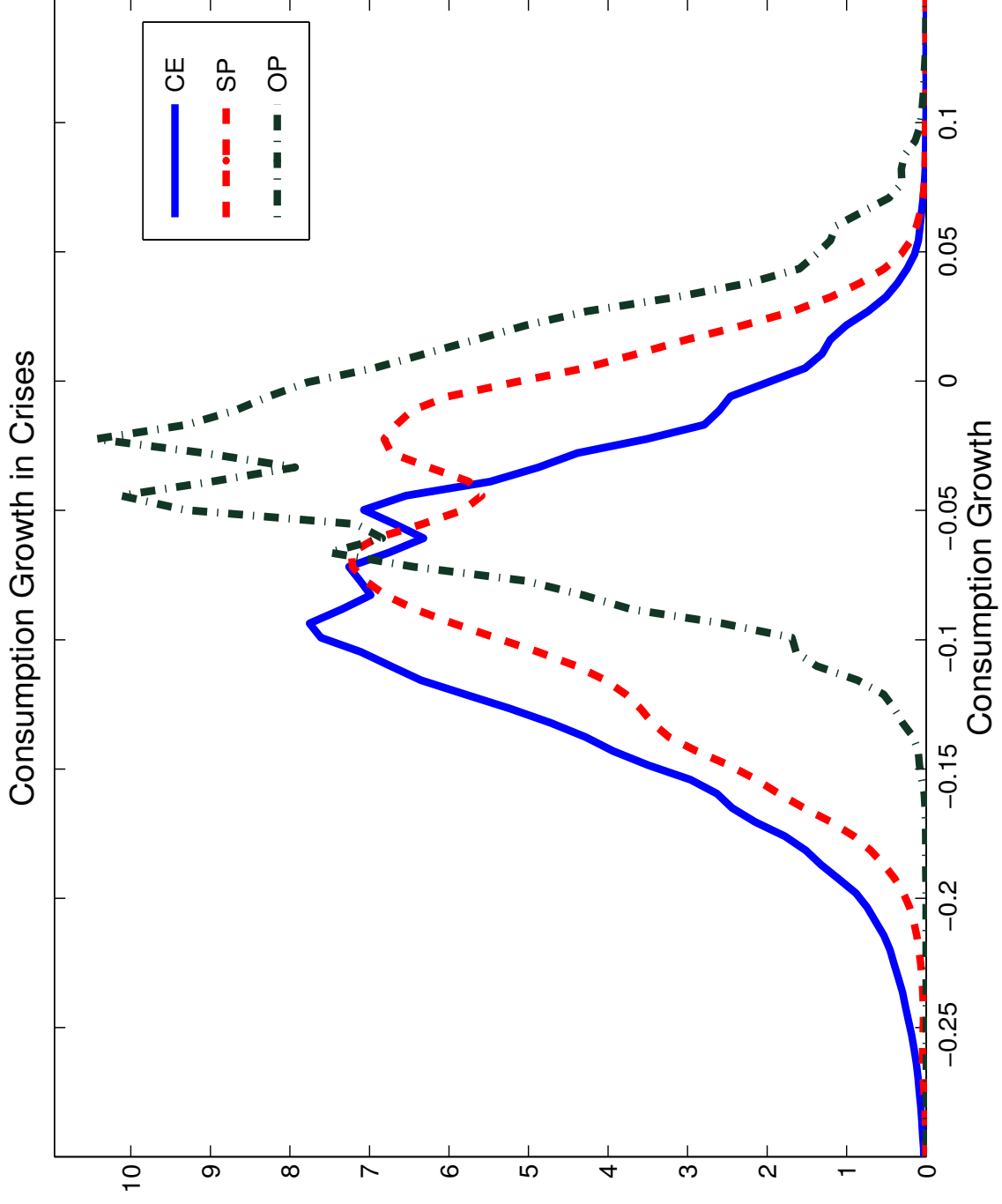
Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The figure plots the ergodic distribution of consumption growth in the period after the constraint was binding. Borrowing decreases from left to right on the x-axis.

A Appendix (for online publication)

A.1 Competitive equilibria

In the competitive equilibrium of the economy with the borrowing constraint and without government intervention (which we also call the "constrained allocation" for brevity), households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T and B_{t+1}. The Lagrangian for this problem is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{1}{1-\rho}C_t^{1-\rho} + \lambda_t \left(B_{t+1} + \frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N \right] \right) + \\ \mu_t \left(Y_t^T + P_t^N Y_t^N - B_{t+1} + (1+r) B_t - C_t^T - P_t^N C_t^N \right), \end{array} \right]$$

with λ_t and μ_t denoting the multipliers on the borrowing constraint and the budget constraint, respectively. The first order conditions of this problem are:

$$C_T : u'(C_t)C_{C^T} = \mu_t, \tag{28}$$

$$C_N : u'(C_t)C_{C^N} = \mu_t P_t^N, \tag{29}$$

$$B_{t+1} : \mu_t = \lambda_t + \beta(1+r) E_t\left[\mu_{t+1}\right]. \tag{30}$$

$$\lambda_t \left\{ B_{t+1} + \frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N \right] \right\} = 0 \tag{31}$$

Combining (28) and (29) we have:

$$\frac{(1-\omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{-\frac{1}{\kappa}}} = P_t^N. \tag{32}$$

This constrained allocation can now be characterized completely by the first order conditions (30), (31) and (32) and the goods market equilibrium conditions:

$$C_t^T = Y_t^T - B_{t+1} + (1+r) B_t, \tag{33}$$

and

$$C_t^N = Y_t^N. \tag{34}$$

A.1.1 Parameter values

The parameter values of the model are set exactly as in Bianchi (2011):

Structural parameters	Values
Elasticity of substitution between tradable and non-tradable goods	$\kappa = .83$
Intertemporal substitution and risk aversion	$\rho = 2$
Credit constraint parameter	$\phi = 0.75758$ 1/
Relative weight of tradable and non-tradable goods	$\omega = 0.31$
Discount factor	$\beta = 0.91$

Exogenous variables	Values
World real interest rate	$r = 0.04$
Steady state endowments	$Y^N = Y^T = 1$

Endowment process

Autocorrelation Matrix
$$\begin{bmatrix} 0.901 & 0.495 \\ -0.453 & 0.225 \end{bmatrix}$$

Variance-Covariance Matrix
$$\begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}$$

Average values in the ergodic distribution	Values
Net foreign assets	$B = -0.91$

1/ This value of ϕ implies a value for $\kappa = .32$ in Bianchi's (2011) notation.

13

A.1.2 Ruling out multiple equilibria

The borrowing constraint can induce multiple equilibria due to the possibility of a self-fulfilling decline in the relative price of nontradables that can reduce the value of the collateral, and the consumption of tradable goods, in a manner compatible with the initial decline in the relative price of nontradables. More formally, by combining the borrowing constraint (4), the budget constraint (3) and the pricing equation we obtain:

$$
C_t^T = B_t(1+r) + \left(1 + \frac{1-\phi}{\phi}\right)\left[Y_t^T + \frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\right] \equiv f\left(C_t^T\right).
$$

When the elasticity of intratemporal substitution is less than 1 (goods are complements), a sufficient condition for unicity is that the derivative of the RHS of this expression with respect to C^T ($f'\left(C^T\right)$), evaluated at the intersection point with the LHS is greater than 1. Indeed, when $B_t(1+r) + \left(1 + \frac{1-\phi}{\phi}\right)Y_t^T < 0$ and $\kappa < 1$, we have that $\lim_{C^T \to 0} f'(C^T) = 0$ and $\lim_{C^T \to \infty} f'(C^T) = \infty$. These conditions, which are satisfied in our calibration, combined with the assumption that $f'\left(C^T\right) > 1$ evaluated at the intersection point, guarantee that the equilibrium is unique—see also the discussion of Jeanne and Korinek (2012).

Another issue that might arise in our model, given our specification of the borrowing constraint, is the possibility that, when the amount that the planner borrows increases, then the relative price of nontradable rises the value of the collateral by more than the increase in B_{t+1}, thus leading to a relaxation of the borrowing constraint. Our calibration also rules out the possibility of such a perverse dynamic.

A.1.3 Unconstrained allocation

In terms of equilibrium conditions, the allocation without the borrowing constraint, which we call the "unconstrained allocation", is fully characterized by the following equations:

$$
u'(C_t^{UE})C_{C^T}^{UE} = \mu_t^{UE}, \tag{35}
$$

$$
u'(C_t^{UE})C_{C^N}^{UE} = \mu_t^{UE}\left(P_t^N\right)^{UE}, \tag{36}
$$

$$
\mu_t^{UE} = \beta\left(1+r\right)E_t\left[\mu_{t+1}^{UE}\right], \tag{37}
$$

along with the goods market equilibrium conditions (7) and (8).[27]

---

[27]See Mendoza (2002) for a comparison of the quantitative properties of the constrained and the unconstrained competitive equilibrium of the model.

A.1.4 First best allocation

1 With perfect access to complete asset markets, an allocation that we call the "first best",
2 households maximize utility subject to their budget constraint state-by-state and period-by-
3 period, expressed in units of tradable consumption.

Let's assume agents can trade a set of state-contingent securities at time 0, which pay
one unit of tradables in a particular state at time t. Let z^t describe the history of shocks at
time t, and let $\pi_0(z^t)$ denotes the time-zero probability of a particular history, with $Q_0(z^t)$
denoting the time-zero price of a security paying one unit of tradables at time t in history
z^t. We denote with $\pi_t(z_{t+1}|z^t)$ the probability of being in state z_{t+1} in time $t+1$ conditional
on a given history at time t. Then the period budget constraint of an individual agent is:

$$\sum_{z_{t+1}} Q_t(z_{t+1}|z^t) B_{t+1}^j (z_{t+1}) + C_t^T + P_t^N C_t^N = Y_t^T \left(z^t\right) + P_t^N Y_t^N \left(z^t\right) + B_t^j.$$

Note here that state-contingent securities can only pay off in terms tradable goods.[28] We
also continue to assume that in our economy there is a lower bound on debt that is strictly
greater than the natural debt limit, $\underline{B} > B^n$. Since nontradable consumption is equal to
its endowment, which is positive in every state of nature, this lower bound guarantees that
tradable consumption has a strictly positive lower bound. Thus, the following constraint
needs to be satisfied:

$$B_{t+1} \left(z^{t+1}\right) \geq \underline{B} \left(z^{t+1}\right)$$

or

$$C_t^T > 0.$$

4 Optimal behavior in this economy can be characterized in terms of the following first
5 order conditions:

$$C_T : (C_t)^{\frac{1}{\kappa}-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} = \mu_t \left(z^t\right), \tag{38}$$

6

$$C_N : (C_t)^{\frac{1}{\kappa}-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} = \mu_t \left(z^t\right) P_t^N, \tag{39}$$

7

$$B_{t+1} \left(z^{t+1}\right) : Q_t(z_{t+1}|z^t) \mu_t \left(z^t\right) = \beta \pi(z_{t+1}|z^t) \mu_{t+1} \left(z^{t+1}\right). \tag{40}$$

---

[28]This is a natural restriction, as agents in different countries can only promise to transfer tradable
goods to each other in different states because, by their nature, non-tradable goods cannot be transferred
internationally. In equilibrium non-tradable consumption must still equal non-tradable output in each state,
for each country.

Combining (38) and (39) we have:

$$\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}} = P_t^N. \tag{41}$$

We can then combine (38) and (39) with the definition of the price index to get:

$$P_t = \left[\omega\left(\frac{(C_t)^{\frac{1}{\kappa}-\rho}\,\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}{\mu_t\left(z^t\right)}\right)^{1-\kappa} + (1-\omega)\left(\frac{(C_t)^{\frac{1}{\kappa}-\rho}(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\mu_t\left(z^t\right)}\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}}.$$

From this expression we obtain

$$\left(\mu_t P_t\right)^{1-\kappa} = \left(C_t\right)^{\frac{(1-\kappa)(1-\rho\kappa)}{\kappa}}\left(C_t\right)^{\frac{\kappa-1}{\kappa}},$$

which becomes

$$\mu_t P_t = C_t^{-\rho}$$

and holds for every state of nature. Since the price of a state contingent asset is common across countries and it is the same as the one that the small open economy faces, we have that

$$Q_t(z_{t+1}|z^t)\mu_t^*\left(z^t\right) = \beta\pi\left(z_{t+1}|z^t\right)\mu_{t+1}^*\left(z^{t+1}\right),$$

where μ^* denote the rest of the world marginal utility of tradeable consumption. It is now easy to obtain

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\frac{P_t^*}{P_{t+1}^*}.$$

Iterating on this condition and defining $\delta_0 \equiv \left(\frac{C_0}{C_0^*}\right)^{-\rho}\frac{P_0^*}{P_0}$, we finally obtain the familiar, complete market risk sharing condition

$$\left(\frac{C_t^*}{C_t}\right)^{-\rho} = \delta_0\frac{P_t^*}{P_t},$$

which links the ratio of the national price levels, or the real exchange rate, to the consumption
differential with the rest of the world.

    Figure 1A plots the lifetime utility or the first best and unconstrained allocations and
shows that the two differs significantly only for high level of initial debt.

A.2 Social planner allocation

A benevolent social planner maximizes (1) subject to the same borrowing constraint (4) that private agents face and the market clearing conditions for tradables and nontradables goods (7) and (8).

In specifying this problem, the equilibrium price of nontradables is determined competitively according to the pricing rule (6) that serves also as a constraint to the planning problem. By substituting the relative price of nontradables, P_t^N in the borrowing constraint (4) with the competitive pricing rule (6) we can write the Lagrangian of the planning problem as

$$
L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{1}{1-\rho} \left(C_t\right)^{1-\rho} + \mu_{1,t}^{SP} \left(Y_t^T - B_{t+1} + (1+r) B_t - C_t^T\right) + \\ + \mu_{2,t}^{SP} \left(Y_t^N - C_t^N\right) + \lambda_t^{SP} \left(B_{t+1} + \frac{1-\phi}{\phi} \left[Y_t^T + \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega Y_t^N}\right)^{\frac{1}{\kappa}} Y^N\right]\right) \end{array} \right],
$$

where $\mu_{1,t}^{SP}$, $\mu_{2,t}^{SP}$ and λ_t^{SP} denote the multipliers and the superscript SP distinguishes them from those in the constrained and unconstrained allocations. The planner must choose the optimal path for C_t^T, C_t^N and B_{t+1}, and the first order conditions for its problem are:

$$
C_T : u'(C_t^{SP})C_{C^T}^{SP} + \lambda_t^{SP}\Sigma_t^{SP} = \mu_{1,t}^{SP}, \tag{42}
$$

$$
C_N : u'(C_t^{SP})C_{C^N}^{SP} = \mu_{2,t}^{SP}, \tag{43}
$$

$$
B_{t+1} : \mu_{1,t}^{SP} = \lambda_t^{SP} + \beta \left(1 + r\right) E_t \left[\mu_{1,t+1}^{SP}\right]. \tag{44}
$$

$$
\lambda_t^{SP} \left\{ B_{t+1}^{SP} + \frac{1 - \phi}{\phi} \left[Y_t^T + P_t^N Y_t^N\right] \right\} = 0 \tag{45}
$$

where $\Sigma_t^{SP} \equiv \frac{1-\phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega}\right)^{\frac{1}{\kappa}-1} \left(Y_t^N\right)^{\frac{\kappa-1}{\kappa}}$.

The key difference between the planning allocation and the competitive one (with the borrowing constraint) follows from examining equations (42) and (28). The planner internalizes the consequences of her/his decisions on P_t^N. When the constraint binds ($\lambda_t^{SP} > 0$), there is an additional benefit in consuming an extra unit tradable consumption, represented by the term $\lambda_t^{SP}\Sigma_t$. This term captures the increase in the price of non-tradable goods associated with the marginal increase in tradable consumption. As we we discuss in the paper, this difference between the two margins has intertemporal implications and affects agents behavior also when the constraint does not bind.

A.3 Optimal policy with a tax on debt

Proof of Proposition 1   *In an economy defined by (1), (4), (12) and (9) with a tax on τ_t^B as the government instrument, the Ramsey optimal policy with τ^B as instrument replicates the social planner allocation (SP). Moreover the optimal policy is time-consistent.*

PROOF: Let's consider first a less restricted version of the Ramsey problem in which the planner maximizes (1) subject to (7) and (8), (17) and (19). This problem corresponds to the social planner one (SP) defined above. The solution of the Ramsey problem for τ^B cannot achieve a higher welfare than the social planner allocation because the Ramsey problem is more restricted than the social planner problem—by equation (18).

We conjecture that the two allocation coincide. To verify this, note that the Euler equation for the social planner problem is

$$u'(C_t^{SP})C_{C_t^T}^{SP} + \lambda_t^{SP}\Sigma_t^{SP} = \lambda_t^{SP} + \beta(1+r)E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP} + \lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]. \tag{46}$$

It is easy to see that, if the Ramsey planner chooses $\left(1 + \tau_t^B\right)$ in equation (18) so that

$$\tau_t^B = (u'(C_t^{SP})C_{C_t^T}^{SP})^{-1}(\lambda_t^{SP}\Sigma_t^{SP} - \beta(1+r)E_t[\lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]), \tag{47}$$

the Euler equations (46) and (30) become identical. It follows that the solution of the Ramsey problem for τ^B and the social planner problem above coincide, and the expression (47) is the Ramsey optimal policy for τ_t^B.

Moreover, since the optimal policy for τ^B decentralizes the social planner problem, which is a recursive problem that can be represented by value function iteration and only depends on the current state $(\{B_t, Y_t^N, Y_t^T\})$, the equilibrium is subgame perfect and the policy rule (47) is time-consistent.

QED.

A.4 Optimal policy with a tax on nontradable consumption

Proof of Proposition 2   *In an economy defined by (1), (4), (12) and (13) with a tax on non-tradable consumption τ_t^N as the government instrument, there exists a policy for τ_t^N that decentralizes the unconstrained allocation. This policy is Ramsey optimal and time-consistent.*

PROOF: For a given state $\{Y_t^N, Y_t^T, B_t\}$, let B_{t+1}^{UE} be the next-period debt and $P_t^{N,UE}$ the current period relative price of non tradable goods in the economy defined by (1) and (3) but without the credit constraint (4)—i.e., in the unconstrained economy satisfying (35)-(37).

Next, let $\hat{P}_t^N$ be the minimum price such that the credit constraint would be met if it were present. Thus:

$$\hat{P}_t^N = \max\left\{0, -\frac{B_{t+1}^{UE} + \frac{1-\phi}{\phi}Y_t^T}{\frac{1-\phi}{\phi}Y_t^N}\right\}.$$

If we set τ^N such that $\hat{P}_t^N(1 + \tau_t^N) \leq P_t^{N,UE}$, then the credit constraint does not bind. In other words, let $\hat{\tau}_t^N = P_t^{N,UE}/\hat{P}_t^N - 1$, then any $\tau_t^N \in (-1, \hat{\tau}_t^N]$ would eliminate the credit constraint ($\lambda_t = 0\ \forall t$) if it were present, and the competitive equilibrium of the economy would coincide with the unconstrained allocation, which eventually converges to the lower debt bound, $\underline{B}$.

Now, in the economy with the credit constraint, the Ramsey planner maximizes (1) subject to (7) and (8), (22), (20), and (21). In this problem, any policy schedule such that $\tau_t^N \in (-1, \hat{\tau}_t^N]$ can achieve an allocation that satisfies the first order conditions (35)-(37) of the unconstrained allocation. Since τ_t^N can affect the allocation only when the constraint binds, but it is neutral when the constraint does not bind, the Ramsey planner can achieve at best the unconstrained allocation. Thus, the tax policy $\tau_t^N \in (-1, \hat{\tau}_t^N]$ is the optimal solution of the Ramsey problem. Moreover, any $\tau_t^N \in (-1, \hat{\tau}_t^N]$ is completely determined by the current state $\{B_t, Y_t^T, Y_t^N\}$ and therefore it is time-consistent.

QED.

A.5   Optimal policy with a tax on tradable consumption

The Ramsey problem when τ_t^T is the policy instrument is as follows.

Definition 4   *For a given $\{B_0\}$ and assuming that $\{Y_t^T\}$ and $\{Y_t^N\}$ are Markov processes with finite, strictly positive support, the Ramsey problem for τ_t^T is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{\beta^t u\left(C_t\right)\right\},$$

subject to the resource constraints

$$C_t^T = Y_t^T - B_{t+1} + (1+r)B_t, \tag{48}$$

$$C_t^N = Y_t^N, \tag{49}$$

the borrowing constraint

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N\right]. \tag{50}$$

the government budget constraint (16) and the first order conditions of the household

$$\frac{u'(C_t)C_{C_t^T}}{1+\tau_t^T} = \lambda_t + \beta\left(1+r\right)E_t\left[\frac{u'(C_{t+1})C_{C_{t+1}^T}}{1+\tau_{t+1}^T}\right].$$ (51)

with

$$\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}} = \frac{P_t^N}{1+\tau_t^T}.$$ (52)

The tax on tradable consumption affects not only the intratemporal relative price in (52), but also the intertemporal allocation of resources in (51). Despite this difference, the next proposition shows that it is possible to find a policy for τ_t^T that replicates the unconstrained allocation like in the case of the nontradable consumption tax τ_t^N.

Proposition 3 *In an economy defined by (1), (3), (15) and (16) with a tax on tradable consumption τ_t^T as the government instrument, there exists a policy for τ_t^T that decentralizes the unconstrained allocation and it is time-consistent.*

PROOF: Let the optimal non-tradable consumption tax be $\hat{\tau}_t^N$. In the Ramsey problem for τ_t^T, if we set $\frac{1}{1+\tau_t^T} = 1+\tau_t^N$ we can achieve the unconstrained allocation, and $\lambda_t \equiv 0 \; \forall t$. However, since τ_t^T affects also the intertemporal allocation of resources (51) we need to show that there is a constant τ_t^T such that the intertemporal margin is not affected.

To do so, we first note that, by imposing $\lambda_t \equiv 0$ and setting τ_t^T so that

$$\frac{1}{1+\tau_t^T} = \frac{\beta(1+r)E_t\left[\frac{u'(C_{t+1}^{UN})C_{C_{t+1}^T}^{UN}}{1+\tau_{t+1}^T}\right]}{E_t[u'(C_{t+1}^{UN})C_{C_{t+1}^T}^{UN}]},$$ (53)

the Euler equations of the Ramsey problem and the unconstrained equilibrium coincide. It follows that the tax rate τ_t^T that satisfies (53) must be constant (otherwise the intertemporal margin would be distorted).

By inspection of the unconstrained allocation, the non-tradable price has a strictly positive lower limit. Therefore there exists $\underline{\tau}^T$ (this is, the lower level of the tax on tradables compatible with the strictly positive lower limit on the relative price of nontradables) such that the borrowing constraint (4) is always satisfied for any $\tau^T \geqslant \underline{\tau}^T$. Thus, any constant tax policy of the form $\tau_t^T \equiv \tau^T \geqslant \underline{\tau}^T$ is an optimal policy such that the competitive equilibrium replicates the unconstrained equilibrium. As $\underline{\tau}^T$ is completely determined by the current state $\{B_t, Y_t^T, Y_t^N\}$ it is time-consistent.

QED.

A.6 Optimal policy with distortionary financing

A.6.1 Time consistency of optimal policy with two instruments

We now prove formally that the Ramsey problem with two distortionary policy instruments is time-consistent.

Ramsey optimal policy with two distortionary policy instruments solves the following problem:

$$(\{B_{t+1}^R\}, \{(\tau_t^N)^R\}, \{(\tau_t^B)^R\}) \doteq \arg \max_{(\{B_{t+1}\}, \{\tau_t^N\}, \{\tau_t^B\})} \sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N), \qquad (54)$$

subject to conditions (7), (8), (20), (24) and (25) for all $t = 0, 1, \cdots$.

The time consistent optimal policy solves the following recursive problem

$$(B_{t+1}^C, (\tau_t^N)^C, (\tau_t^B)^C) \doteq \arg \max_{(B_{t+1}, \tau_t^N, \tau_t^B)} U(C_t^T, C_t^N) + \beta V^C(B_{t+1}) \qquad (55)$$

subject to conditions (7), (8), (20), (24) and (25) at time t. Here $V^C(\cdot)$ is the household value function under the time consistent optimal policy, i.e.

$$V^C = \sum_{t=0}^{\infty} \beta^t U(C_t^{TC}, C_t^{NC})$$

where $\{C_t^{TC}\}$ and $\{C_t^{NC}\}$ are sequences of tradeable and nontradable consumptions based on the time consistent optimal policy. Note that the state of economy at time t is B_t, the current level of debt. Hence, the value function depends solely on B_t. We want to establish that, in our economy, the Ramsey optimal policy is time consistent, i.e. $B_{t+1}^R = B_{t+1}^C, \tau_t^{NR} = \tau_t^{NC}, \tau_t^{BR} = \tau_t^{BC}$.

To prove this, we shall take the following steps. First, we show that this is the case in a three-period version of these two problems. Second, we look at a four-period case and show that this can be reduced to the 3-period case. Next we show that we can always reduce an n-period case to a $n-1$-period one for any $n > 4$. This establishes, by induction, that in any finite-period version of our model economy the two policy regimes coincide. Finally, under the auxiliary assumption that the period utility function and the marginal utility of consumption are bounded in the feasible set, we prove that Ramsey optimal policy in the

finite-period model converges to Ramsey optimal policy in a infinite-horizon version of our economy.

Three-period model We start by examining the 3-period version of the original Ramsey optimal policy problem:

$$(\{B_1^{R'}, B_2^{R'}\}, \{\tau_0^{NR'}, \tau_1^{NR'}\}, \{\tau_0^{BR'}, \tau_1^{BR'}\}) \doteq$$

$$\arg \max_{(\{B_1, B_2\}, \{\tau_0^N, \tau_1^N\}, \{\tau_0^B, \tau_1^B\})} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2), \tag{56}$$

subject to (7), (8), (20), (24) and (25) for all $t = 0, 1, \cdots$.

It is easy to see that the only potential source of difference between the two policy regimes comes from the Euler equation (24). In fact, when we optimizes at time $t = 1$ in the time-consistent regime, we do not take into account that the choice of B_2 affects the Euler equation at time $t = 0$,

$$U_{C_0^T}(1 + \tau_0^B) = \lambda_0 + \beta(1 + r)U_{C_1^T},$$

since from (7) B_2 affects $U_{C_1^T}$.

However this can result in differences between the two policy regimes only if B_2 affects $U(C_0^T)$ and $U(C_1^T, C_1^N) + \beta V^C(B_2)$ in opposite ways. Specifically, in order for the following two problems

$$\max_{(\{B_1, B_2\}, \{\tau_0^N, \tau_1^N\}, \{\tau_0^B, \tau_1^B\})} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2) \tag{57}$$

$$= \max_{(B_1, \tau_0^N, \tau_0^B)} U(C_0^T, C_0^N) + \beta \left(\max_{(B_2, \tau_1^N, \tau_1^B)} U(C_1^T, C_1^N) + \beta^2 V^C(B_2) \right),$$

to coincide, it is sufficient that the following derivatives have the same sign:

$$\frac{\partial U_0(B_2)}{\partial B_2} \text{ and } \frac{\partial U_1(B_1, B_2)}{\partial B_2}, \tag{58}$$

where

$$U_0(B_2) \doteq \max_{(B_1, \tau_0^N, \tau_0^B)} U(C_0^T, C_0^N),$$

subject to (7), (8), (20), (24) and (25) at time $t = 0$, and

$$U_1(B_1, B_2) \doteq \max_{(\tau_1^N, \tau_1^B)} U(C_1^T, C_1^N) + \beta V^C(B_2)$$

subject to (7), (8), (20), (24) and (25) at time $t = 1$.

If this restriction holds, the maximization of $U_1(B_1, B_2)$ with respect to B_2, yields the same optimal value of B_2 that maximizes $U_0(B_2)$. Therefore the maximization can be done in a step-wise way (which gives the time consistent optimal policy) for the Ramsey program on the left hand side of the equality (57).

Thus, in order to show that in our economy Ramsey optimal policy is time consistent we need to establish (58). To do this, we are going to show that both $U_0(B_2)$ and $U_1(B_1, B_2)$ are decreasing functions of B_2, given B_1. In fact, it is straightforward to see that the function $U_0(B_2)$ is a decreasing function of B_2, since if the household knows that in period 2 she can borrow more, she is able to consume more in period 1, and through the Euler equation (24), she can also consume more in period 0.

Next we want to show that $U_1(B_1, B_2)$ is also a decreasing function of B_2, for given B_1. Let B_2^* be the borrowing level in the competitive equilibrium without the borrowing constraint or any tax intervention. So $U_1(B_1, B_2)$ must achieves its maximum at B_2^*. Therefore $U_1(B_1, B_2)$ decreases for any $B_2 \geq B_2^*$. We shall show that $B_2^C \geq B_2^*$ in the optimal plan that maximize $U_1(B_1, B_2)$ subject to (7), (8), (20), (24) and (25) for $t = 1$.

We know from our optimal policy analysis on the individual tax instruments, that the optimal policy is such that $\tau_1^{NC} \leq 0$ and $\tau_1^{BC} \leq 0$. If the borrowing constraint is not binding, we have from the Euler equation (24) that

$$U_{C_1^T}(1 + \tau_1^B) = \beta(1 + r)U_{C_2^T}.$$

And if $\tau_1^B < 0$ and the $B_2 < B_2^*$, we would have

$$U_{C_1^T}(1 + \tau_1^B) < U_{C_1^{T*}} = \beta(1 + r)U_{C_2^{T*}} < \beta(1 + r)U_{C_2^T},$$

which is a contradiction.[29] Therefore we conclude that if the borrowing constraint is not binding, $B_2^C \geq B_2^*$.

If the constraint is binding, from the Euler equation (24) we have that $\lambda > 0$. Suppose that $B_2' \geq B_2^*$ is optimal in the economy without the borrowing constraint. We want to show that the optimal policy in the economy with the borrowing constraint has $B_2 \geq B_2'$. Suppose this is not the case. Then we would have

$$U_{C_1^T}(1 + \tau_1^B) - \lambda < U_{C_1^{T'}}(1 + \tau_1^B) = \beta(1 + r)U_{C_2^{T'}} < U_{C_2^T},$$

---

[29]The first inequality comes from $C_1^T > C_1^{T*}$ and the fact that $U_{C_1^T}$ is a decreasing function of C_1^T. The second inequality comes from $C_2^T < C_2^{T*}$ and the same logic.

17 which again contradicts the Euler equation (24).[30] Therefore we must have that $B_2^C \geq B_2^{C'} \geq$

18 B_2^*. Combining the previous two arguments, it follows that $U_1(B_1, B_2)$ is also a decreasing

1 function of B and hence has the same sign of $U_0(B_2)$, which proves that 58 holds.

Finite-period model Let us now look first at the case of a four-period model. We will show that this case can be reduced to the 3-period model above. In a four-period version of our model, the Ramsey program solves the following problem

$$\max_{\left(\{B_i\}_{i=1}^3, \{\tau_i^N\}_{i=0}^2, \{\tau_i^B\}_{i=0}^2\right)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 U(C_2^T, C_2^N) + \beta^3 V^C(B_3),$$

2 subject to (7), (8), (20), (24) and (25) for $t = 1, \cdots, 3$.

Now note first that

$$U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N)$$

is decreasing in B_3 by the same reasoning as in the 3-period model, and that

$$U(C_2^T, C_2^N) + \beta V^C(B_3)$$

is also decreasing in B_3, since $B_3 \geq B_3^*$ where B_3^* is the competitive equilibrium borrowing level without the borrowing constraint or tax interventions. Therefore we have that

$$\max_{\left(\{B_i\}_{i=1}^3, \{\tau_i^N\}_{i=0}^2, \{\tau_i^B\}_{i=0}^2\right)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 U(C_2^T, C_2^N) + \beta^3 V^C(B_3)$$

$$= \max_{\left(\{B_i\}_{i=1}^2, \{\tau_i^N\}_{i=0}^1, \{\tau_i^B\}_{i=0}^1\right)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^{C'}(B_2),$$

where

$$V^{C'}(B_2) = \max_{B_3, \tau_2^N, \tau_2^B} U(C_2^T, C_2^N) + \beta V^C(B_3)$$

3 subject to (7), (8), (20), (24) and (25) for $t = 3$. Thus, we reduced a four-period model

4 into a three-period model. It follows that the Ramsey optimal policy is time consistent in a

5 four-period version of our model.

6 By using the same method, we can always reduce an n-period model into $n-1$ period

7 model for any $n > 4$. By induction, therefore, we showed that the Ramsey optimal policy

1 for any finite-period version of our economy is time consistent.

---

[30]The inequalities follow from the same reasonings in the case of a nonbinding borrowing constraint.

Infinite-horizon model   If we can establish the convergence of the Ramsey optimal policy problem for a finite-period version of our model to an infinite-period version, we will have established that Ramsey optimal policy is time consistent for (54). To do so, we need an additional assumption, i.e. that both $U(\cdot)$ and $U_{C^T}(\cdot)$ are bounded in the feasible set.

Define now the following mapping $T : \mathcal{F}_b \to \mathcal{F}_b$, where $\mathcal{F}_b$ is the set of bounded continuous function defined on $[\underline{B}, 0] \times \mathbb{R}^+$,

$$T(V)(B, \mu) \doteq \min_{\gamma \geq 0} \max_{B', \tau^B} U(C^T, C^N) - \mu(1 + r)U_{C^T} + \gamma(U_{C^T} - \lambda) + \beta V(B', \gamma),$$

subject to (7), (8), (20), (24) and (25).

From the assumption that both $U(\cdot)$ and $U_{C^T}(\cdot)$ are bounded, it follows that λ is bounded. Following Marcet and Marimon (2011), we also conclude that T is a contraction mapping.

In addition, we note that $T^n(V)(B, 0)$ is the welfare function of a Ramsey optimal plan for a n-period economy with $V(\cdot)$ as the final period utility. Therefore from a standard contraction mapping argument we have that

$$V^*(\cdot) \doteq \lim_{n \to \infty} T^n(V)(\cdot)$$

is well defined and is uniformly converging. $V^*(\cdot)$ will be the fixed point of the contraction mapping and is the welfare function of the infinite-period economy under the Ramsey optimal policy.

By the uniform convergence of the welfare function, the finite-period Ramsey optimal policy converges to the infinite-period Ramsey optimal policy. Therefore we established that the Ramsey optimal policy for (54)is time consistent.

QED

A.6.2   Three distortionary policy instruments

We now focus on the case in which all three distortionary policy tools are available to the policy maker (see (26)).

Suppose that the triplet of policy tools $(\tau_t^N, \tau_t^T, \tau_t^B)$ can completely remove the borrowing constraint (4). The Euler equation for this economy would be:

$$\frac{1 + \tau_t^B}{1 + \tau_t^T}\mu_t = \beta(1 + r)E_t\frac{\mu_{t+1}}{1 + \tau_{t+1}^T}. \tag{59}$$

Remember now that the Euler equation for the unconstrained economy is:

$$\mu_t^{UE} = \beta \left(1 + r\right) E_t \left[\mu_{t+1}^{UE}\right]. \tag{60}$$

By comparing (59) and (60), we can see that in order to replicate the unconstrained equilibrium the triplet of policy tools $(\tau_t^N, \tau_t^T, \tau_t^B)$ must satisfy:

$$\frac{1 + \tau_t^B}{1 + \tau_t^T} = \frac{E_t \frac{\mu_{t+1}}{1 + \tau_{t+1}^T}}{E_t \left[\mu_{t+1}^{UE}\right]}. \tag{61}$$

In addition, from the government budget constraint, we need to have

$$\tau_t^T \left(C_t^T\right)^{UE} + \tau_t^N \left(C_t^N\right)^{UE} + \tau_t^B B_{t+1}^{UE} = 0. \tag{62}$$

And from the borrowing constraint, we must have that

$$B_{t+1}^{UE} \geq -\frac{1 - \phi}{\phi} \left[Y_t^T + \left(P_t^N\right)^{UE} Y_t^N \frac{1 + \tau_t^T}{1 + \tau_t^N}\right]. \tag{63}$$

To find the tax policy $\{\tau_t^B, \tau_t^T, \tau_t^N\}$ that solves (61) to (63) we proceed recursively as follows. Denote the stochastic steady state level of debt by $\underline{B}$, and by B_0 the level of debt in the unconstrained equilibrium at which the constraint would become binding exactly in the constrained economy. Now define $B_t = B^{UE}(B_{t-1})$, where $B^{UE}(\cdot)$ is the policy function in the unconstrained equilibrium. From this policy function, we can obtain $B_0 > B_1 > \cdots > B_t > B_{t+1} > \cdots > \underline{B}$, so that $\{B_k\}$ is a debt trajectory in the unconstrained solution starting from B_0.

Starting from $k = 0$, for any $B \in (B_1, B_0]$, we can compute

$$\frac{1 + \tau^T(B)}{1 + \tau^N(B)} = -\frac{B^{UE}(B) + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} \left(P_t^N\right)^{UE}(B) Y_t^N}$$

from (63). Let's set $\tau_0^T(B) \equiv 0$ in that interval and use the expression above to obtain $\tau_0^N(B)$. The value of $\tau_0^B(B)$ in the $(B_1, B_0]$ interval can then be determined by the government budget constraint (62).

Next, consider $k = 1$ and the associated interval $(B_2, B_1]$. Since we have already determined the value of $\tau_0^T(B)$ and $\tau_0^B(B)$ for $B \in (B_1, B_0]$, by using (61), we can obtain the value of $\tau_1^T(B)$. Again by assuming the borrowing constraint (61) is binding exactly, we can determine the value of $\tau_1^N(B)$. Last, by using the government budget constraint (62) we can

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3   determine the value of $\tau_1^B(B)$ and update to $k = 2$.

4   By iterating recursively, we can always find the tax policy that replicates the uncon-

5   strained solution in an economy with the borrowing constraint.

6   QED

7   A.7   Extensions

8   In this section we provide details of the extensions discussed in the paper.

9   A.7.1   Imperfect exchange rate intervention

Let us consider a situation in which both the capital control tax τ^B and the non-tradable consumption tax τ^N are available to the government, and the budget is balanced with a lump-sum tax T. However, the non-tradable consumption tax τ^N can be used only some of the times, depending on the realization of an exogenous random variable $\varepsilon_t \in \{0, 1\}$ that is known to the government at the beginning of each period t. This random loss of access to τ^N could capture imperfect credibility of exchange rate policy or limited availability of revenue to implement the subsidy. With such an imperfect form of exchange rate intervention, the household budget constraint becomes

$$C_t^T + (1 + \varepsilon_t \tau_t^N)P_t^N C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau_t^B) + (1 + r)B_t;$$

the government budget now is

$$\varepsilon_t \tau_t^N P_t^N C_t^N + \tau_t^B B_{t+1} = T_t;$$

and the non-tradable pricing equation now is

$$\frac{(1 - \omega)^{\frac{1}{\kappa}}(C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}(C_t^T)^{-\frac{1}{\kappa}}} = (1 + \varepsilon_t \tau_t^N)P_t^N.$$

10 In order to study this case, we proceed in two steps. We first design the optimal policy for

11 τ_t^N conditional on the realization of ε_t and a given capital control tax τ^B. We then optimize

12 over τ^B taking into account that the optimal policy for τ_t^N might not be always available.

        Similarly to what we found in proposition 2 in the paper, optimal policy for τ_t^N can undo the collateral constraint. Indeed, it is straightforward to see that regardless of the policy for

τ^B, the time-consistent optimal policy for τ^N is

$$\tau_t^N \in \left\{ \begin{array}{ll} \left(-1, \hat{\tau}_t^N \stackrel{\circ}{=} \frac{P_t^{N,TC(\tau^B)}}{\hat{P}_t^N} - 1 \right], & \varepsilon_t = 1, \\ \text{any,} & \varepsilon_t = 0. \end{array} \right\}$$

Here we have denoted with $P_t^{N,TC(\tau^B)}$ the non-tradable price under any time-consistent policy for τ^B. Also

$$\hat{P}_t^N = \max \left\{ 0, -\frac{B_{t+1}^{TC(\tau^B)} + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} Y_t^N} \right\},$$

13 where $B_{t+1}^{TC(\tau^B)}$ denotes borrowing under the same time-consistent policy of τ^B. Essentially,
14 under this policy for τ^N, the government can remove the borrowing constraint.

We then proceed with the second step and study the optimal policy for τ^B taking into account that the optimal policy for τ_t^N might not be always available. To do so, we transform the optimization problem into an equivalent one in which there is only τ^B, the budget balancing lump-sum tax T, and a modified version of the credit constraint:

$$(1 - \varepsilon_t) \left(B_{t+1} + \frac{1 - \phi}{\phi} (Y_t^T + P_t^N Y_t^N) \right) \geqslant 0.$$

This version of the borrowing constraint takes into account that, when exchange rate policy is available, i.e. $\varepsilon_t = 1$, the constraint can be removed. A line of argument similar to the one used in the proof of Proposition 1 in the paper can show that there exists the following time-consistent optimal policy for τ_t^B:

$$\tau_t^B = (u'(C_t^{SP(\varepsilon)}) C_{C_t^T}^{SP(\varepsilon)})^{-1} ((1 - \varepsilon_t) \lambda_t^{SP(\varepsilon)} \Sigma_t^{SP(\varepsilon)} - \beta(1 + r) E_t[(1 - \varepsilon_{t+1}) \lambda_{t+1}^{SP(\varepsilon)} \Sigma_{t+1}^{SP(\varepsilon)}]),$$

1 and it coincides with the Ramsey optimal policy. Here we denoted $SP(\varepsilon)$ as a social planner
2 problem in which the credit constraint binds at time t only when $\varepsilon_t = 0$. And this optimal
3 policy τ_t^B achieves the allocation corresponding to $SP(\varepsilon)$. Thus, this shows that, with
4 imperfect credibility, prudential capital controls are part of the optimal policy design, like
5 in the case in which we have distortionary financing analyzed in the paper. Notice however
6 that, here, the optimal τ_t^B depends on the likelihood that $\varepsilon_t = 0$: the higher the probability
7 that exchange rate policy is not available, the higher the level of τ_t^B.

A.7.2 Borrowing constraints with pre-tax income

Here we show that, when the borrowing constraint is expressed in terms of post-tax income, and the policy tools available to the Ramsey planner is τ^B with $\tau_t^B B_{t+1} = T_t$, the allocation under optimal policy for this instrument is identical to the one in which the borrowing constraint is defined in terms of pre-tax income and the instrument is τ^N with $\tau_t^N P_t^N C_t^N = T_t$. Rewrite the borrowing constraint as

$$
\begin{aligned}
B_{t+1} &\geqslant -\frac{1-\phi}{\phi}\left[Y_t^T + \left(\tilde{P}_t^N\right)Y_t^N - T_t\right] = \\
&= -\frac{1-\phi}{\phi}\left[Y_t^T + \left(\tilde{P}_t^N\right)Y_t^N - \tau_t^B B_{t+1}\right]
\end{aligned}
\tag{64}
$$

where

$$
\tilde{P}_t^N = \frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}
$$

Now, if the borrowing constraint depends on pre-tax income and we have $\tau_t^N P_t^N C_t^N = \tau_t^B B_{t+1}$, we can write the borrowing constraint as

$$
\begin{aligned}
B_{t+1} &\geqslant -\frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N\right] = \\
&= -\frac{1-\phi}{\phi}\left[Y_t^T + \frac{\tilde{P}_t^N Y_t^N}{(1+\tau_t^N)}\right] \\
&= -\frac{1-\phi}{\phi}\left[Y_t^T + \left(\tilde{P}_t^N\right)Y_t^N - \frac{\tau_t^N \tilde{P}_t^N Y_t^N}{(1+\tau_t^N)}\right]
\end{aligned}
\tag{65}
$$

where

$$
\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}} = \tilde{P}_t^N = P_t^N(1+\tau_t^N)
$$

Comparing (64) and (65) we can see that the two coincide since

$$
\frac{\tau_t^N \tilde{P}_t^N Y_t^N}{(1+\tau_t^N)} = \tau_t^N P_t^N C_t^N = \tau_t^B B_{t+1}.
$$

A.8 Numerical solution methods

Here we describe how we compute the different equilibria numerically. We start by rewriting the competitive equilibrium of the model with the borrowing constraint. We can summarize

this equilibrium with the following set of nonlinear functional equations:

$$
\begin{aligned}
\mu\left(B, Y^T, Y^N\right) &= \beta\left(1+r\right) E\left[\mu\left(B'\left(B, Y^T, Y^N\right), Y^{T\prime}, Y^{N\prime}\right)\right] \\
&\quad + \max\left\{\lambda\left(B, Y^T, Y^N\right), 0\right\}^2 \\
\mu\left(B, Y^T, Y^N\right) &= \left(C^{\frac{\kappa-1}{\kappa}}\right)^{(1-\rho)\frac{\kappa}{1-\kappa}-1} \omega^{\frac{1}{\kappa}} C^T\left(B, Y^T, Y^N\right)^{-\frac{1}{\kappa}} \\
C^T\left(B, Y^T, Y^N\right) &= \left(1+r\right) B + Y^T - B'\left(B, Y^T, Y^N\right) \\
P^N\left(B, Y^T, Y^N\right) &= \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{Y^N}{C^T\left(B, Y^T, Y^N\right)}\right)^{-\frac{1}{\kappa}} \\
\max\left\{-\lambda\left(B, Y^T, Y^N\right), 0\right\}^2 &= B'\left(B, Y^T, Y^N\right) + \frac{1-\phi}{\phi} Y^T + P^N\left(B, Y^T, Y^N\right) Y^N,
\end{aligned}
$$

where

$$
C^{\frac{\kappa-1}{\kappa}} \equiv \left[\omega^{\frac{1}{\kappa}}\left(C^T\left(B, Y^T, Y^N\right)\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}}\left(Y_t^N\right)^{\frac{\kappa-1}{\kappa}}\right].
$$

We then convert the complementary slackness conditions for the borrowing constraint in these equations into a nonlinear equation, following Garcia and Zangwill (1981).

A.8.1   The constrained and unconstrained competitive equilibrium

Given an initial guess for the marginal utility of tradable consumption tomorrow $\mu^0\left(B', Y^{T\prime}, Y^{N\prime}\right)$, the set of nonlinear functional equations above can be solved at each point in the state space $\left(B, Y^T, Y^N\right)$ to obtain an updated function $\mu^1\left(B, Y^T, Y^N\right)$. This process is then iterated to convergence. We use a cubic spline to approximate the $\mu^0\left(B', Y^{T\prime}, Y^{N\prime}\right)$ function at values of B' that are not on the grid for B. We obtain the lifetime utility in the competitive equilibrium using the following Bellman equation:

$$
V^{CE}\left(B, Y^T, Y^N\right) = \frac{1}{1-\rho} C^{1-\rho} + \beta E\left[V^{CE}\left(B'\left(B, Y^T, Y^N\right), Y^{T\prime}, Y^{N\prime}\right)\right].
$$

The allocation corresponding to the unconstrained competitive equilibrium is computed in a similar fashion, except that the complementary slackness condition is omitted.

A.8.2   The social planning problem

The solution of the social planning problem solves the following standard dynamic programming problem:

$$
V^{SP}\left(B, Y^T, Y^N\right) = \max_{C^T, B', P^N}\left\{\frac{1}{1-\rho} C^{1-\rho} + \beta E\left[V^{SP}\left(B'\left(B, Y^T, Y^N\right), Y^{T\prime}, Y^{N\prime}\right)\right]\right\}
$$

subject to

$$
\begin{aligned}
C^T + B' &\leq (1+r)B + Y^T \\
B' &\geq -\frac{1-\phi}{\phi}\left(Y^T + P^N Y^N\right) \\
P^N &= \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(\frac{Y^N}{C^T}\right)^{-\frac{1}{\kappa}}.
\end{aligned}
$$

Again, we approximate the value function with a cubic spline and solve the constrained optimization problem using feasible sequential quadratic programming with analytical derivatives.

A.8.3 Markov-Perfect optimal policy

To compute the Ramsey optimal control program with two instruments we exploit time-consistent nature of the problem and use the method proposed by Benigno et al. (2012). That method is related to Klein, Krusell, and Rios-Rull (2009): the main difference being that the algorithm that we use does not require that the policy functions are differentiable (which in general would not hold in our environment due to the occasionally-binding constraint) but only that they are continuous.

The optimal policy problem for τ_N and τ_B is also solved iteratively. The current government solves the following problem

$$
V^{OP}\left(B, Y^T, Y^N\right) = \max_{\tau_N, \tau_B, C^T, P^N, B', \mu, \lambda}\left\{\frac{1}{1-\rho}C^{1-\rho} + \beta E\left[V^{OP}\left(B'\left(B, Y^T, Y^N\right), Y^{T\prime}, Y^{N\prime}\right)\right]\right\}
$$

subject to

$$
\begin{aligned}
(1+\tau_B)\mu &= \beta(1+r)E\left[\mu\left(B'\left(B, Y^T, Y^N\right), Y^{T\prime}, Y^{N\prime}\right)\right] + \max\{\lambda, 0\}^2 \\
\mu &= \left(C^{\frac{\kappa-1}{\kappa}}\right)^{(1-\rho)\frac{\kappa-1}{\kappa}-1}\omega^{\frac{1}{\kappa}}\left(C^T\right)^{-\frac{1}{\kappa}} \\
C^T &= (1+r)B + Y^T - B' \\
P^N &= \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(\frac{Y^N}{C^T}\right)^{-\frac{1}{\kappa}}\frac{1}{1+\tau_N} \\
\max\{-\lambda, 0\}^2 &= B' + \frac{1-\phi}{\phi}\left(Y^T + P^N Y^N\right) \\
0 &= \tau_N P^N Y^N + \tau_B B'\left(B, Y^T, Y^N\right).
\end{aligned}
$$

We then guess both the continuation value function and the future marginal utility function, solve the optimization problem using feasible sequential quadratic programming with analytical derivatives, and then update both functions to convergence.[31] Both functions are approximated with cubic splines.

We set a large number of grid points in the B dimension (1550), with most of them concentrated at the lower end of the debt range where the constraint may bind. The joint process for (Y^T, Y^N) is approximated as a Markov chain with 49 states (7 in each dimension) using the method of Gospodinov and Lkhagvasuren (2013). Invariant distributions were produced using the nonstochastic method from Young (2010), except for the frequency of crises which are estimated using a simulated sample of $10,000,000$ observations.

A.8.4 Welfare calculations

To compute the welfare equivalents, we solve the following functional equation:

$$\widetilde{V}^{CE}\left(B, Y^T, Y^N; \chi\right) = \frac{1}{1-\rho}C^{1-\rho} + \beta E\left[\widetilde{V}^{CE}\left(B'\left(B, Y^T, Y^N\right), Y^{T\prime}, Y^{N\prime}; \chi\right)\right];$$

where χ is a proportional increment to tradable consumption, and the decision rules are those from the competitive equilibrium. We use 200 grid points for χ, evenly-spaced. We then solve the following nonlinear equations for $\chi\left(B, Y^T, Y^N\right)$:

$$V^{SP}\left(B, Y^T, Y^N\right) = \widetilde{V}^{CE}\left(B, Y^T, Y^N; \chi\right),$$

to obtain the welfare gain from moving to the SP allocation;

$$V^{OP}\left(B, Y^T, Y^N\right) = \widetilde{V}^{CE}\left(B, Y^T, Y^N; \chi\right),$$

to obtain the welfare gain from moving to the OP allocation; and

$$V^{UA}\left(B, Y^T, Y^N\right) = \widetilde{V}^{CE}\left(B, Y^T, Y^N; \chi\right)$$

to obtain the welfare gain from moving to the unconstrained allocation. These equations are solved using the Brent's method, with linear interpolation between grid points for χ.

---

[31]We use analytical derivatives, particularly for the continuation value function, as numerical derivatives produce solutions that are "choppy" for the tax variables (but not the other endogenous variables).

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