Resolving the Missing Deflation Puzzle

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Post-crisis policy challenges and implications for macro modelling
Key observations during the Great Recession:

- Extraordinary contraction in GDP but only small drop in inflation.

Source: Christiano, Eichenbaum and Trabandt (2015, AEJ: Macro)
Motivation

- Small drop in inflation referred to as the “missing deflation puzzle”:

- John C. Williams (2010, p. 8): “The surprise [about inflation] is that it’s fallen so little, given the depth and duration of the recent downturn. Based on the experience of past severe recessions, I would have expected inflation to fall by twice as much as it has.”
Recent work emphasizes role of financial frictions to address the missing deflation puzzle:


We propose an alternative resolution of the puzzle:

- Importance of nonlinearities in price and wage-setting when the economy is exposed to large shocks.
What We Do

- Study inflation and output dynamics in linearized and nonlinear formulations of the standard New Keynesian model.

- **Key feature:** real rigidity to reconcile *macroevidence* of low Phillips curve slope and *microevidence* of frequent price re-setting.

- Study implications for:
  - Propagation of shocks
  - Nonlinear Phillips curves
  - Unconditional distribution of inflation (skewness)
**Framework**

  - Monopolistic competition and Calvo sticky prices and wages.
  - Fixed aggregate capital stock.
  - ZLB constraint on nominal interest rate.

Outline

- Benchmark model
- Parameterization
- Results
- Analysis in estimated model
- Conclusions
Model: Final Good Firms

- Competitive firms aggregate intermediate goods $Y_{f,t}$ into final good $Y_t$ using technology $\int_0^1 G \left( \frac{Y_{f,t}}{Y_t} \right) df = 1$.

- Following Dotsey-King (2005) and Levin-Lopez-Salido-Yun (2007):

  \[
  G \left( \frac{Y_{f,t}}{Y_t} \right) = \frac{\omega_p}{1 + \psi_p} \left[ (1 + \psi_p) \left( \frac{Y_{f,t}}{Y_t} \right) - \psi_p \right]^{\frac{1}{\omega_p}} + \frac{1 + \psi_p - \omega_p}{1 + \psi_p}
  \]

- $\psi_p < 0$: Kimball (1995), $\psi_p = 0$: Dixit-Stiglitz.

- Kimball aggregator: demand elasticity for intermediate goods increasing function of relative price.
  - Dampens firms’ price response to changes in marginal costs.
Demand Curves

- Dixit-Stiglitz ($\psi_p = 0$)
- Kimball ($\psi_p = -3$)
- Kimball ($\psi_p = -12$)

Relative Demand $y_f / y$, log-scale

Relative Price $P_f / P$, log-scale

Demand Curves (Levin, Lopez-Salido and Yun, 2007)
Optimal Pricing Decisions in Simplified Model

**Kimball: Profit Functions**
- $p_{f_{opt}} = 0.9970$
- $p_{f_{opt}} = 1.0000$
- $p_{f_{opt}} = 1.0045$
- Marginal Cost: -5%
- Marginal Cost: +5%

**Dixit-Stiglitz: Profit Functions**
- $p_{f_{opt}} = 0.950$
- $p_{f_{opt}} = 1.000$
- $p_{f_{opt}} = 1.050$
- Marginal Cost: -5%
- Marginal Cost: +5%

**Kimball: Optimal Relative Price**
- % Change in $p_{f_{opt}}$
- % Change in Marginal Cost
- Nonlinear Model
- Linearized Model

**Dixit-Stiglitz: Optimal Relative Price**
- % Change in $p_{f_{opt}}$
- % Change in Marginal Cost
- Nonlinear Model
- Linearized Model
Model: Intermediate Good Firms

- Continuum of monopolistically competitive firms $f$
  - Hire workers and rent capital; production technology $Y_{f,t} = K^\alpha_f N^{1-\alpha}_{f,t}$
  - Calvo sticky prices: optimal price set with probability $1 - \xi_p$, otherwise simple updating $\tilde{P}_t = (1 + \pi) P_{t-1}$.

- Fixed aggregate capital stock $K \equiv \int K_f df$. 
Model: Households

- Household $j$ preferences:
  \[
  \max E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left\{ \ln C_{j,t} - \omega \frac{N_{j,t}^{1+\chi}}{1+\chi} \right\}
  \]
  \[\zeta_t - \text{discount factor shock.}\]

- Budget constraint:
  \[
P_t C_{j,t} + B_{j,t} = W_{j,t} N_{j,t} + R^K_{j} K_{j} + (1 + i_{t-1}) B_{j,t-1} + \Gamma_{j,t} + A_{j,t}
  \]
Model: Households

- Standard Euler equation

\[ 1 = \beta E_t \left( \delta_{t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \frac{C_t}{C_{t+1}} \right) \]

\[ \delta_{t+1} \equiv \frac{\xi_{t+1}}{\xi_t} \text{ where } \delta_t \text{ follows an AR(1) process.} \]

- Calvo sticky wages (same conceptual setup as for sticky prices previously discussed).
Aggregate resource constraint:

\[ C_t = Y_t \leq \frac{1}{p_t^*(w_t^*)^{1-\alpha}} K^\alpha N_t^{1-\alpha} \]

where \( p_t^* \) and \( w_t^* \) are Yun’s (1996) aggregate price and wage dispersion terms.
Taylor rule:

\[
1 + i_t = \max \left\{ 1, (1 + i) \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y_{pot}} \right)^{\gamma_x} \right\}
\]

where \( Y_{pot} \) denotes flex price-wage output.

Taylor rule in “linearized” model:

\[
i_t - i = \max \left\{ -i, \gamma_\pi (\pi_t - \pi) + \gamma_x x_t \right\}
\]
Solve linearized and nonlinear model using Fair-Taylor (1983, ECMA):

- Two-point boundary value problem.

- Solution of nonlinear model imposes certainty equivalence (just as linearized model solution does by definition).

- Solution algorithm traces out implications of not linearizing equilibrium equations.

- Use Dynare for computations: ‘perfect foresight solution’/‘deterministic simulation’.

- Robustness: global stochastic solution, see Lindé-Trabandt (2018).
Parameterization

- **Price setting:**
  
  - $\zeta_p = 0.67$ (3 quarter price contracts), $\phi_p = 1.1$ (10% markup).
  
  - $\psi_p = -12.2$ (Kimball) and $\beta = 0.9975$ (discounting) so that
    \[ \kappa_p \equiv \frac{(1-\zeta_p)(1-\beta \zeta_p)}{\zeta_p} \frac{1}{1-\phi_p \psi_p} = 0.012 \]
    
    in $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_p \hat{mc}_t$
    

- **Wage setting:**
  
  - $\zeta_w = 0.75$, $\phi_w = 1.1$ and $\psi_w = -6$ (approx. estimate in estimated model).
Parameterization

- Labor share = 0.7 ($\alpha = 0.3$), linear labor disutil. ($\chi = 0$)

- Steady state inflation 2 percent, nominal interest rate 3 percent.

- Taylor rule: $\gamma_\pi = 1.5$, $\gamma_x = 0.125$.

- $\delta_t$ follows AR(1) with $\rho = 0.95$
Results: Effects of a Discount Factor Shock

- Follow ZLB literature: assume negative demand shock hits economy.
- Discount factor shock $\delta_t$ rises by 1 percent before gradually receding.
Figure 2: Impulse Responses to a 1% Discount Factor Shock

Panel A: ZLB Not Imposed

Nonlinear Model
Linearized Model

Panel B: ZLB Imposed
Results: Intuition

- What accounts for the muted inflation response in the nonlinear version of the New Keynesian model?

- Key: nonlinearity of Kimball aggregator and firms’ demand curve.
  - Recall: demand elasticity *falls* when relative price of a firm falls.
  - Thus, firms’ ability increase demand by cutting price is limited.
  - Large price cut results in lower profits because demand would increase only by little $\rightarrow$ little incentive for firms to cut prices a lot.

- Nonlinearity stronger the deeper the recession. Linearization produces large approximation errors.
Next, do stochastic simulations of linearized and nonlinear model using discount factor shocks.

Subject both models to long sequence of discount factor shocks:

- \( \delta_t - \delta = 0.95 (\delta_{t-1} - \delta) + \sigma \varepsilon_t \) with \( \varepsilon_t \sim N(0, 1) \)

Set \( \sigma \) such that prob(ZLB) = 0.10 in both models.
Figure 3: Stochastic Simulation of Nonlinear and Linearized Model

Panel A: Nonlinear Model

Panel B: Linearized Model
Results: Phillips Curve

Price Phillips Curve (Kimball)

- Inflation Rate (APR)
- Negative Output Gap (% of potential - actual GDP)

- Nonlinear Model
- Linearized Model
Results: Wage Phillips Curve

Wage Phillips Curve (Kimball)

- **Nonlinear Model**
- **Linearized Model**

Wage Phillips Curve

- **Wage Inflation Rate (APR)**
- **Negative Output Gap (%: potential - actual GDP)**
Analysis in Estimated Model

- Assess robustness in CEE/SW workhorse model.

- Key model features:
  - Nominal price and wage stickiness
  - Kimball aggregation in prices and wages
  - Endogenous capital accumulation
  - Habit persistence and investment adjustment costs
  - Variable capital utilization
Analysis in Estimated Model

- Estimate linearized model on standard macro data (SW 2007)
  - Output, consumption, investment, hours, price inflation, wage inflation, federal funds rate.
  - Same seven shocks as in SW (2007).

- Estimate 27 parameters
  - Calibrate price and wage stickiness parameters ($\xi_p = 0.66$ and $\xi_w = 0.75$) and markups ($\phi_p = \phi_w = 1.1$).
  - Estimate Kimball parameters $\psi_p$ (post. mean -12.5) and $\psi_w$ (post. mean -8.3).
Next, we aim to examine the model’s ability to shed light on the ‘missing deflation puzzle’.

Subject nonlinear and linearized model to risk premium shock:

- Risk premium shock as in Smets-Wouters (2007). Bondholding FOC:
  \[ \lambda_t = \beta E_t \lambda_{t+1} \frac{\epsilon_{RP,t} R_t}{\Pi_{t+1}}. \]

- \( \epsilon_{RP,t} \) elevated for 16 quarters before gradually receding. Increase \( \epsilon_{RP,t} \) such that both models deliver a fall in output as in the data.

- Compare resulting paths of model and data for inflation.
Analysis in Estimated Model: Great Recession

Notes: Data and model variables expressed in deviation from no-Great Recession baseline.
Data from Christiano, Eichenbaum and Trabandt (2015)
Next, study the implications of the nonlinear and linearized model for the Phillips curve.

Simulate the model for each of the seven exogenous processes using the estimated model parameters.
Analysis in Estimated Model: Phillips Curves

Risk Premium Shocks Only

Monetary Policy Shocks Only

Gov. Cons. Shocks Only

Technology Shocks Only

Investment Shocks Only

Price Markup Shocks Only

Wage Markup Shocks Only

All Shocks

- Inflation (APR) vs. Negative Output Gap (% of potential GDP)
Analysis in Estimated Model: Inflation Densities

Figure 8: Densities of Data vs. Stochastic Model Simulations

- Core PCE Inflation
- Wage Inflation (Hourly Earnings)
- Real GDP Growth
- Federal Funds Rate

Data (1965Q1-2007Q4) Linearized Medium-Sized Model Nonlinear Medium-Sized Model
Conclusions


- Our nonlinear NK model with Kimball aggregation resolves the ‘missing deflation puzzle’ while the linearized version fails to do so.

- Our nonlinear model generates nonlinear Phillips curves and reproduces the skewness of price and wage inflation observed in post-war U.S. data.

- All told, our results caution against the common practice of using linearized models when the economy is exposed to large shocks.
Additional Slides
Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007):

$$G \left( \frac{Y_{f,t}}{Y_t} \right) = \frac{\omega_p}{1 + \psi_p} \left[ \left( 1 + \psi_p \right) \left( \frac{Y_{f,t}}{Y_t} \right) - \psi_p \right] \frac{1}{\omega_p} + \frac{1 + \psi_p - \omega_p}{1 + \psi_p}$$

- $G (\cdot)$ strictly concave and increasing function.

- $\omega_p = \frac{1 + \psi_p}{1 + \phi_p \psi_p} \phi_p$, $\phi_p > 1$ gross price markup, $\psi_p \leq 0$ Kimball param.

- Special case: $\psi_p = 0 \rightarrow$ Dixit-Stiglitz.
Kimball FOC’s:

- First order conditions can be written as:

  Demand Curve : \( \frac{Y_{f,t}}{Y_t} = \frac{1}{1 + \psi_p} \left( \frac{P_{f,t}}{P_t \vartheta_t^p} \right)^{\varepsilon_p} + \frac{\psi_p}{1 + \psi_p} \)

  Agg. Price Index : \( 1 = \int_0^1 \left( \frac{P_{f,t}}{P_t \vartheta_t^p} \right)^{\varepsilon_p} df \)

  Zero Profits : \( \vartheta_t^P = 1 + \psi_p - \psi_p \int_0^1 \frac{P_{f,t}}{P_t} df \)

- \( \varepsilon_p = \frac{\phi_p (1 + \psi_p)}{1 - \phi_p} \). \( \vartheta_t^P \) – Lagr.-multiplier on aggregator constraint.

- Special case: \( \psi_p = 0 \) \( \rightarrow \) standard Dixit-Stiglitz expressions:

  \( \frac{Y_{f,t}}{Y_t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\frac{\phi_p}{\phi_p - 1}}, P_t = \left( \int P_{f,t}^{\frac{1}{1-\phi_p}} df \right)^{1-\phi_p} \)
Linearized Price and Wage Phillips Curves

- **Price Inflation Phillips curve:**

\[
\hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{(1 - \tilde{\zeta}_p) (1 - \beta \tilde{\zeta}_p)}{\tilde{\zeta}_p} \frac{1}{1 - \phi_p \psi_p} \hat{mc}_t
\]

\(\phi_p > 1\) (gross price markup), \(\psi_p \leq 0\) (Kimball parameter prices).

- **Wage Inflation Phillips curve:**

\[
\hat{\Pi}^w_t = \beta E_t \hat{\Pi}^w_{t+1} + \frac{(1 - \tilde{\zeta}_w) (1 - \beta \tilde{\zeta}_w)}{\tilde{\zeta}_w} \frac{1}{1 - \phi_w \psi_w} (\hat{mrs}_t - \hat{w}_t)
\]

\(\phi_w > 1\) (gross wage markup), \(\psi_w \leq 0\) (Kimball parameter wages).
Set $\psi_p = \psi_w = 0 \rightarrow$ Dixit-Stiglitz. Set $\zeta_p = \zeta_w = 0.9$ to keep slopes of linearized Phillips curves unchanged.

Kimball aggregator crucial for muted inflation response.
Benchmark Model: Large vs. Small Shocks

### Benchmark Shock Size

#### Price Phillips Curve (Kimball)

- **Nonlinear Model**
- **Linearized Model**

#### Wage Phillips Curve (Kimball)

- **Nonlinear Model**
- **Linearized Model**

### Small Shock Size

#### Price Phillips Curve (Kimball)

- **Nonlinear Model**
- **Linearized Model**

#### Wage Phillips Curve (Kimball)

- **Nonlinear Model**
- **Linearized Model**
Use global projection method to solve stochastic nonlinear model:
- Discretize state space. Linear interpolation/extrapolation for points not exactly on grid nodes.
- Evaluate expectation terms using trapezoid rule.

Stochastic processes for consumption demand shock, \( \nu_t \):

\[
(\nu_t - \nu) = 0.80 (\nu_{t-1} - \nu) + \sigma \varepsilon_{\nu,t}, \varepsilon_{\nu,t} \overset{iid}{\sim} \mathcal{N}(0, 1)
\]

Parameter \( \sigma_{\nu} \) tuned such that probability of being the ZLB is roughly 10 percent in each quarter (Nakata, 2016).
Stochastic Model Solution (Lindé-Trabandt, 2018)

Impulse Responses to a Consumption Demand Shock

1. Output Gap
2. Inflation (APR)
3. Nominal Interest Rate (APR)
4. Real Interest Rate (APR)
5. Potential Real Interest Rate (APR)
6. Cons. Demand Shock
Results indicate that implications of uncertainty in **nonlinear model** are quantitatively small/negligible.

- Reason: flat Phillips curve and Kimball aggregator.
- Hence, we focus on the deterministic nonlinear solution.

By contrast, shock uncertainty affects **linearized model** solution to a much greater degree.
Analysis in Est. Model: Phillips Curves Cond. on 8Q ZLB

Inflation and output gap in deviation from baseline -- annualized percentage points and percentage points, respectively. Baseline is a demand shock driven deep recession that triggers a liquidity trap where the ZLB is expected to bind for 8 quarters. Random shocks from estimated model hit in the first quarter when ZLB binds in the baseline. Inflation and output gap shown one year after random shocks have hit.