Housing and the secular decline in real interest rates

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Oesterreichische Nationalbank

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*The content of these slides reflects the views of the authors and not necessarily those of the OeNB.
The decline in real long-run interest rates

- Laubach/Williams (2003): Decrease for the United States since 1980: around 3%.

**Figure 4.** Changes in AE $R^*$ and Trend Growth, 1971–2016

- Natural rate of interest
- Trend growth
Explanations for the decline in real interest rates

• Demographic aging increased the need for old age provision (Eggertson et al., 2019; Auclert et al., 2021)
• Increasing income inequality and the “saving glut of the rich” (Mian et al., 2020)
• Global saving glut and safe assets shortage (Bernanke, 2005; Caballero et al., 2017)
• Calibrated models confirm the importance of these factors:
  - Mankiw (2022): A simple Solow model with only $s$ and $g + n$
  - Platzer/Peruffo (2022): A large-scale model
This paper

- Study a model that includes an asset in fixed supply (land/housing).
- The model (based on Piketty, 2011) is stylized in order to provide intuition and (under certain conditions) closed-form solutions: deterministic, real-term, closed economy, focus on steady-state comparisons.
- Main questions: Does the existence of housing . . .
  - dampen the fall in interest rates (by absorbing excess savings)?
  - change the relative importance of the different channels?
  - help to explain other stylized facts?
Other important long-run trends

- **Private wealth-income ratios:**
  
  300% (1980) → 540% (2018)

- **Share of housing wealth:**
  
  36% (1970) → 53% (2015)

- **Other:** Inheritance flows, saving rates, volume of mortgages, inequality, ...

*Sources:* Piketty/Zucman (2014), Wealth Inequality Report (2018), Bauluz/Novokmet/Schularick (2022)*
Benchmark model

- I consider a structure with **four groups** \((r, om, oo, w)\):
  - Renters
  - Owner-occupiers with mortgage
  - Outright owners (without a mortgage)
  - Top 1\% (outright owners, higher bequest motive)

- **Model assumptions:**
  - Owners with mortgages continuously refinance their purchases (no transaction costs)
  - Outright owners inherit a certain house and pass it on to their children who do the same.

  Short-cut for:
  - Houses that people are not *allowed* to sell (trusts etc.)
  - Houses that people are not *willing* to sell (“old family property” etc.)
  - Sluggishness over the lifecycle (“aging in place” etc.)
Equilibrium interest rate

- **Asset supply** (=Wealth demand):
  - Physical capital $K_t$
  - Houses $H_t$ ($= H_t^r + H_t^{om} + H_t^{oo} + H_t^w$)
  - Government bonds $D_t$

- **Asset demand** (=Wealth supply):
  - By households with a life-cycle and a bequest motive

- Write the wealth-to-income ratio as $\beta \equiv \frac{\text{Wealth}_t}{\text{NDP}_t}$ (and $\beta_Z = \frac{Z_t}{\text{NDP}_t}$ for asset $Z_t$)

- The equilibrium interest rate $r^*$ solves:

  $$\beta = \beta_K + \beta_{Hr} + \beta_{Hom} + \beta_D = \tilde{\beta}$$

  Wealth Demand

  Wealth Supply
Demography

- **Continuous-time OLG model.** Individuals …
  - become adults at age $A$,
  - are employed until retirement at age $R$,
  - die at age $D$,
  - receive a bequest at age $I$ (with $A \leq I \leq R$),
  - receive a pension with net replacement rate $\rho$ after retirement.

- Each **cohort** born in time $x$ has a size $N^x = N^0 e^{nx}$ and includes a continuum $i \in [0, 1]$ of individuals.
Warm-glow model

Lifetime utility $V^j(A)$ for group $j \in \{r, om, oo, w\}$ consists of:

- **Intratemporal** function:
  \[
  u^j(a) = \frac{(\eta^j h^j(a))^{\gamma} (c^j(a))^{1-\gamma}}{(\gamma)^{\gamma} (1-\gamma)^{1-\gamma}} \quad \rightarrow \quad \text{determines } P^r_{st} \text{ and } P^o_{st}
  \]

- **Intertemporal** function:
  \[
  U^j(A) = \left\{ \frac{\int_A^D e^{-\theta(a-A)} \left(u^j(a)\right)^{1-\sigma} \, da}{\int_A^D e^{-\theta(a-A)} \, da} \right\}^{\frac{1}{1-\sigma}}
  \]

- **Intergenerational** function:
  \[
  V^j(A) = (1 - s^j_B) \log \left(U^j(A)\right) + s^j_B \log \left(w^j(D)\right)
  \]

- Determines $\tilde{\beta}$ depending on two crucial savings motives:
  - A life-cycle motive (for net replacement rate $\rho < 1$):
  - A bequest motive (for $s^j_B > 0$).
Production

- Output of “normal” (non-housing) goods and services:
  \[ Y_{Nt} = K_t^\alpha (A_t L_t)^{1-\alpha} \]

- Productivity \( A_t \) grows at rate \( g \), labor supply \( L_t \) at rate \( n \).
- Factor markets are competitive.
- The net return on capital:
  \[ r_{kt} = \alpha \frac{Y_{Nt}}{K_t} - \delta_k \]

- The capital-to-income ratio in steady state (with \( r_{kt} = r_k \)):
  \[ \beta_{K}^N \equiv \frac{K_t}{Y_{Nt}} = \frac{\alpha}{r_k + \delta_k} \]
Housing 1

- Housing supply: \( \overline{H}_t = \overline{H}_t^r + \overline{H}_t^o \)
  - \( \overline{H}_t^r \) . . . rented stock, \( \overline{H}_t^o = \overline{H}_t^{om} + \overline{H}_t^{oo} + \overline{H}_t^w \) . . . owned stock
  - The housing stocks grow at rate \( n \).

- Rental housing:
  - The rental housing-wealth-to-income ratio:

\[
\beta_{Hr}^N = \frac{P_{ht}^r \overline{H}_t^r}{Y_{Nt}} = \frac{P_{st}^r \overline{H}_t^r}{Y_{Nt}} \frac{1}{r_h + \delta_h - g}
\]

- \( P_{st}^r \): rent (service price), \( P_{ht}^r \): purchasing price
- \( r_{ht} \): Rate of return on investments into rental housing:

\[
r_{ht} = \frac{P_{st}^r}{P_{ht}^r} - \delta_h + \frac{\dot{P}_{ht}^r}{P_{ht}^r} \rightarrow P_{ht}^r = \frac{P_{st}^r}{r_{ht} + \delta_h - \frac{\dot{P}_{ht}^r}{P_{ht}^r}}
\]

- In the steady state \( \frac{\dot{P}_{ht}^r}{P_{ht}^r} = g \) and \( r_{ht} = r_h \).
Housing 2

- **Owned housing:**
  - The owned housing-wealth-to-income ratio:

\[
\beta_{Ho}^N \equiv \frac{P^o_{ht} H_{t}^o}{Y_{Nt}} = \frac{P^o_{st} H_{t}^o}{Y_{Nt}} (r_m \delta_h - g)
\]

- \(P^o_{st}\): imputed rent (shadow service price), \(P^o_{ht}\): purchasing price
- Assumption: All home purchases are fully financed by mortgages at the rate \(r_{mt}\) and there is continuous re-financing (no transaction costs).
- Imputed rent: \(P^o_{st} = (r_{mt} + \delta_h - \frac{\dot{P}^o_{ht}}{P^o_{ht}}) P^o_{ht}\).
Summary: Equilibrium interest rate

• Equilibrium condition:

\[
\begin{align*}
\beta &= \beta_K + \beta_{Hr} + \beta_{Hom} + \beta_D = \beta \\
\text{Wealth Demand} &
\end{align*}
\]

\[
\text{Wealth Supply}
\]

• Wealth Demand:

\[
\begin{align*}
\beta_K &= \frac{\alpha}{r_k + \delta_k}, \beta_D \quad \text{(public debt, assumed as given)} \\
\beta_{Hr} &= \frac{P_{st}H_t^r}{Y_{Nt}}, \beta_{Ho} = \frac{P_{st}^oH_t^o}{Y_{Nt}} \\
\end{align*}
\]

• Equilibrium interest rates:

\[
\begin{align*}
r^* &= \frac{1}{\beta N} (\beta_K r_k + \beta_{Hr} r_h + \beta_{Ho} r_m + \beta_D r_d) \\
r_h &= r_k - \xi_h, \quad r_m = r_k - \xi_m, \quad r_d = r_k - \xi_d
\end{align*}
\]
Calibration

- Focus on steady-state comparisons between an “initial” situation (around 1980) and a “current situation” (around 2018).
- The values refer to the group of advanced countries
- **Demographic and economic parameters**
  
  Initial: \( g = 3.0\% \), \( n = 1.0\% \), \( D = 75 \), \( R = 65 \), \( \rho = 70\% \), \( \beta_N^N = 20\% \)
  Current: \( g = 1.8\% \), \( n = 0.5\% \), \( D = 82 \), \( R = 63 \), \( \rho = 60\% \), \( \beta_N^N = 70\% \)
- **Bequest motive**
  
  Initial: \( s_B \) and \( s_{B_{top1\%}} \) such that \( \beta = 350\% \) and \( \frac{\text{Wealth of top 1\%}}{\text{Total Wealth}} = 28\% \)
  Current: \( s_{B_{top1\%}} \) changes such that \( \frac{\text{Wealth of top 1\%}}{\text{Total Wealth}} = 35\% \)
- **Risk discounts**
  
  Initial: \( \xi_h = 0\% \), \( \xi_m = 2\% \), \( \xi_d = 5\% \), Current: \( \xi_m = 3\% \)
- **Population shares**
  
  Renters: 50% → 40%, Mortgage owners: 25% → 35%
  Outright owners: Pop. share constant, share of owned houses ↑
Numerical results

<table>
<thead>
<tr>
<th>Case</th>
<th>r</th>
<th>$\beta$</th>
<th>$\beta_K$</th>
<th>$\frac{\beta_H}{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline model (4 groups)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Initial</td>
<td>9.6%</td>
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Results 1

- Decrease in $r$ by 3.9 pp (from 9.6% to 5.7%)
- Increase in $\beta$ by 250 pp (from 350% to 599%)
- Increase in the $\beta_H/\beta$ by 8 pp (from 46% to 54%)
- Results are broadly in line with the observed data.
  - $\Delta r \approx 3$ pp
  - $\beta$ from 300%-350% (1970-80) to 500%-550% (2015-18)
  - $\beta_H/\beta$ from 36% (1970) to 53% (2015)
  - Sources: Piketty/Zucman, 2014; Alvaredo et al., 2018; Bauluz et al., 2022

- Most of the increase in $\beta$ is due to housing wealth (only moderate increase in $\beta_K$).

- Comparison to the case without (outright) owners:
  - Without (outright) owners the model implies a reduction in the share of housing wealth.

- Comparison to the case without housing:
  - Existence of housing increases $r$ and $\beta$ (even though $\beta_K \downarrow$).
## Results 2

<table>
<thead>
<tr>
<th>Case</th>
<th>$r$</th>
<th>$r_k = r_h$</th>
<th>$r_m$</th>
<th>$r_d$</th>
<th>$\beta$</th>
<th>$\frac{\beta_H}{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>9.6%</td>
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<td>5.3%</td>
<td>350%</td>
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</tr>
<tr>
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<td>7.2%</td>
<td>4.2%</td>
<td>2.2%</td>
<td>599%</td>
<td>54%</td>
</tr>
</tbody>
</table>

- **Interest rates:**
  - Mortgage rate $r_m$: 8.3% → 4.2%
  - Interest rate on safe assets: 7.3% → 3.3%
  - Interest rate on gov. bonds: 5.3% → 2.2%
  - Return on housing (rented & owner-occupied): 9.2% → 4.8%

- **Mortgages-to-GDP** ratio increases from 36% to 74%
  (in the data from 25% to 65% (Jordà et al., 2016)).

- **Inheritance flow** increases from 6.6% to 9.2%
  (in the data from around 6% to around 11%).
Comparison to the literature

Decomposition of the decline in the real interest rate in Rachel/Summers (2019), Platzer/Peruffo (2022) and this paper.

<table>
<thead>
<tr>
<th>Variable</th>
<th>RS '19</th>
<th>PP '22</th>
<th>This paper (4 Groups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP growth ((g))</td>
<td>-1.8</td>
<td>-1.00</td>
<td>-1.12</td>
</tr>
<tr>
<td>Pop. growth ((n))</td>
<td>-0.6</td>
<td>-0.25</td>
<td>-0.39</td>
</tr>
<tr>
<td>Longer retirement ((D))</td>
<td>-1.1</td>
<td>-0.46</td>
<td>-0.78</td>
</tr>
<tr>
<td>Length of working life ((R))</td>
<td>-0.1</td>
<td>–</td>
<td>-0.07</td>
</tr>
<tr>
<td>Replacement rate ((\rho))</td>
<td>–</td>
<td>–</td>
<td>-0.13</td>
</tr>
<tr>
<td>Inequality ((s^w_B \text{ and } d^w_y))</td>
<td>-0.7</td>
<td>-0.70</td>
<td>-1.90</td>
</tr>
<tr>
<td>Public Debt ((\beta^N_D))</td>
<td>+3.6</td>
<td>+0.31</td>
<td>+0.29</td>
</tr>
<tr>
<td>Interactions</td>
<td>-1.1</td>
<td>-0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>Other factors</td>
<td>–</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td>Total</td>
<td>-1.8</td>
<td>-2.16</td>
<td>-3.95</td>
</tr>
</tbody>
</table>
Additional implications

- Positive correlation of outright ownership rates with aggregate wealth and with housing wealth share.  

- The evidence in Fagereng et al. (2019) about “capital gains savers” is also compatible with the assumption of sticky outright owners.

- The model offers an explanation for a divergent trend in the rates of gross and net savings.
Conclusions

- A model that includes land/housing is compatible with a decline in the interest rate due to aging, increasing inequality and a slowdown in productivity growth.
- In order to also explain the rise in the housing share it is necessary to introduce additional elements like the existence of owner occupiers.
- The full model is not only (broadly) in line with the developments of $r$, $\beta$ and $\beta_H/\beta$ but also with other important magnitudes like: the inheritance flows, the volume of mortgages, the importance of capital gain savers.
Appendix
National accounting

$$\text{NDP}_t =$$

$$Y_{nt} + (P^r_{st} \overline{H}^r_t + P^o_{st} \overline{H}^o_t) + (\dot{P}^r_{ht} \overline{H}^r_t + \dot{P}^o_{ht} \overline{H}^o_t) - \delta_k K_t - \delta_h \left( P^r_{ht} \overline{H}^r_t + P^o_{ht} \overline{H}^o_t \right)$$

\(\text{GDP}^{NA}_t\)

\(\text{GDP}_t = \text{Haig-Simmons national income}\)

- The Haig-Simmons definition of national income is the theory-consistent concept. Its use has been suggested, e.g., by Robbins (2018) and Fagereng et al. (2019).
- Note that empirically often \(\frac{\text{NDP}_t}{Y_{nt}} \approx 1 \rightarrow \beta_t \approx \beta^N_t\).
- Since: \(\beta_t \equiv \frac{\text{Wealth}_t}{\text{NDP}_t} = \frac{\text{Wealth}_t}{Y_{nt}} \frac{Y_{nt}}{\text{NDP}_t} = \beta^N_t \frac{Y_{nt}}{\text{NDP}_t} \approx \beta^N_t\).
Rates of returns on housing vs. equity

TABLE VII
Real Rates of Return on Equity and Housing

<table>
<thead>
<tr>
<th>Country</th>
<th>Full sample</th>
<th></th>
<th></th>
<th>Post-1950</th>
<th></th>
<th></th>
<th>Post-1980</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Equity</td>
<td>Housing</td>
<td>Equity</td>
<td>Housing</td>
<td>Equity</td>
<td>Housing</td>
<td>Equity</td>
</tr>
<tr>
<td>Average, unweighted</td>
<td>6.67</td>
<td>7.26</td>
<td>8.30</td>
<td>7.47</td>
<td>10.78</td>
<td>6.43</td>
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</tr>
<tr>
<td>Average, weighted</td>
<td>7.12</td>
<td>6.72</td>
<td>8.19</td>
<td>6.40</td>
<td>9.08</td>
<td>5.50</td>
<td></td>
</tr>
</tbody>
</table>

Source: Jordà et al., 2019

- Depending on the sample a risk discount between 1% and 4% seems reasonable (average between $\xi_h$ and $\xi_m$).
- Eichholtz et al. (2021) and Chambers et al. (2021) find lower (risk-adjusted) returns to rental housing for Amsterdam, Paris and Oxbridge colleges than Jordà et al. (2019).
Intuition for the behavior of the housing share

Without outright owners:

\[
\frac{\beta_H}{\beta_K} = \frac{1}{Y_{nt}} \left( \frac{P_{st}^r H_t^r}{r_h + \delta_h - g} + \frac{P_{st}^o H_t^{om}}{r_m + \delta_h - g} \right) \cdot \frac{\alpha}{r_k + \delta_k}.
\]

Simple example:

- Assumptions:
  \[ P_{st}^r H_t^r = \gamma Y_{Lt}^r, \quad P_{st}^o H_t^{om} = \gamma Y_{Lt}^o \]
  \[ r_k = r_h = r_m = g + n, \quad \delta_k = \delta_h = 0 \]

\[ \frac{\beta_H}{\beta_K} = \frac{\gamma(1-\alpha)}{\alpha} \frac{g+n}{n} \]

\[ \frac{\partial \left( \frac{\beta_H^N}{\beta_K^N} \right)}{\partial t} = \frac{\gamma(1-\alpha)}{\alpha} \frac{g}{n} \left( \frac{\dot{g}}{g} - \frac{\dot{n}}{n} \right) \]
Intuition for the behavior of the housing share 2

With outright owners:

\[
\frac{\beta_H}{\beta_K} = \frac{\gamma(1 - \alpha)}{\alpha} \left[ \frac{r_k + \delta_k}{r_h + \delta_h - g} + \left(1 - \kappa_N^r + \frac{\kappa_H^{od} - \kappa_N^{od}}{1 - \kappa_H^{od}} \right) \frac{r_k + \delta_k}{r_m + \delta_h - g} \right],
\]

\(\kappa_N^j\) ... population size of group \(j\), \(\kappa_H^j\) ... size of the housing stock.

- Implications:
  - \(\beta_{HN}^j \equiv \frac{p_{ht}^j \bar{H}_t^j}{\gamma_{Nt}} = \frac{p_{st}^j \bar{H}_t^j}{\gamma_{Nt}} \frac{1}{r_j + \delta_h - g} \).
  - Note: \(p_{st}^j \bar{H}_t^j = \gamma E_t^j\). A change in \(\bar{H}_t^j\) has no effect on \(\beta_{HN}^j\).
  - Now: Assume \(\bar{H}_t^{od} \uparrow\) and \(\bar{H}_t^{om} \downarrow\). Then \(p_{st}^o\uparrow\) such that \(p_{st}^o \bar{H}_t^{om}\) stays the same. But then \(p_{st}^o \bar{H}_t^{od} \uparrow\).
Outright owners

- **Wide variation** across countries: 15%-25% (AT, DE, NL) to > 75% (Eastern Europe).
- In the **UK** the share increased from 37% (1980) to 41% (2018).
- Positive correlation of outright ownership rates with aggregate wealth and with housing wealth share.

**Add. implications**

(a) $\beta_H/\beta$ vs. outright owners

(b) $\beta$ vs. outright owners
Capital gains savers (Fagereng et al., 2019)

Figure 1: Saving rates across the wealth distribution.

(a) Net vs. gross savings

(b) Portfolio shares

Notes: The figure displays the mean portfolio share in percent of total assets across the wealth distribution, by percentile group. Safe assets is the sum of deposits, bonds, and informal loans. Debt is the sum of private debt and debt held indirectly via private firms. Public equity is the sum of directly-held stocks, stock funds, and stocks held indirectly via private firms. Private business is the book value of private firms, taking out public stocks and debt.
Inheritance flow and aggregate savings ratio

- **Bequest:**
  - The inheritance flow is defined as $b_{yt}^N = \frac{B_t}{Y_{Nt}}$.
  - The ratio of *financial bequests* moves only weakly from 5.9% to 7.1% in the 4 groups model. But this excludes the bequest of the directly owned housing stock.
  - The mortality rate is given by $m = \frac{n}{e^{n(D-A)}-1}$ which is about 1.36% in both situations. The *inherited directly owned housing stock* amounts to: $m \times \beta_{Hod}^N$. This adds 0.7pp to $b_{yt}^N$ (initial) and 2.1pp (today).
  - For data on some countries: [See](#).

- **Savings:**
  - The aggregate gross savings rates (including all capital gains) is $\bar{s} = 29.9\%$ (initial) which increases to $\bar{s} = 33.9\%$ (today).
  - If one excludes capital gains from savings and from GDP: $\bar{s} = 28.2\% \rightarrow 31.4\%$
  - The net savings rate: $\bar{s}^{net} = 15.3\% \rightarrow 15.1\%$ (with capital gains); $\bar{s}^{net} = 12.9\% \rightarrow 11\%$ (w/o capital gains)
Inheritance flows in Europa, 1900-2010

Source: Alvaredo et al., 2017