Practical DSGE models

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Introduction

- Policymakers somewhat dissatisfied with their models. Lack of flexibility? Lack of forecasting power? Lack of appropriate storytelling?


- Agent-based /new economic thinking model failed so far to deliver tractable frameworks to be used in policy institutions.

- Future: Less structured models? HANKs?

- Alternative: ”Practical DSGEs”: use more data for estimation and analysis, exploit institutional information, use time series blocks to account for missing features.
History of macro modeling


- Constructed from national account identities.
- Specify demand relationships with delay adjustments.
- No supply side, no capital stock, no expectations.
- Focus on multiplier effects of exogenous fiscal-monetary changes.

- Add supply side considerations.

- Dynamics describe adjustments of today’s value to long run (equilibrium) targets.

- Introduce the notions of gaps and of inflation dynamics.

- Introduce (adaptive) expectations; financial assets (Tobin’s Q, etc.)


- Construct the steady state from static optimization problems; calibrate it to the data; add dynamics if needed (reverse process relative to 2G).

- Equations functions of "deep parameters". Use policy rules.

- Focus analysis on the effects of shocks (rather than changes in exogenous variables) on gaps.


- Use dynamic optimization to develop decision rules. Use frictions to create slow adjustments to shocks.


- General equilibrium (stock-flow consistency). Central role for Euler equation and Phillips curves.

- Long run and short fluctuations jointly considered.

- Tight corset. Good for story telling; difficult short run forecasting performance; no role for institutional knowledge or non-rational behavior.
5G: Large heterogeneity of views.


- Stripped down version of 4G models with non-structural or ad-hoc features: COMPASS (2013); MAJA,(2019).

Practical DSGEs


- Core DSGE structure.

- Add data, non-core features, institutional information, etc. via measurement equations.

- Use large information set for estimation and forecasting (macro, micro, institutional, etc.)

- Flexibility; more precise parameter and latent variable estimates; smaller forecast uncertainty; more robust inference.
Basic idea

• DSGE core (4G Model):

\[ x_t = A(\theta)x_{t-1} + B(\theta)e_t \]
\[ y_t = C(\theta)x_{t-1} + D(\theta)e_t \]  
\[ z_t = S[x'_t, y'_t]' = F(\theta)z_{t-1} + G(\theta)e_t, \text{ } n \times 1 \text{ matrix; } S= \text{ selection matrix} \]  

• There could be latent objects (potential output, NAIRU, R*) in \((x_t, y_t)\).

• \(e_t\) could include permanent disturbances (generating balance growth) and transitory disturbances (generating deviation from balance growth).
Measurement equations:

\[ w_t = G(\lambda)z_t + F(\psi, q_t) + u_t \]  \hspace{1cm} (2)

where \( w_t = [w_{1t}, w_{2t}] \) is a vector observables, \( w_{1t} \) data counterpart of model variables, \( w_{2t} \) proxies/variables not in the model, \( u_t \) is measurement error vector, \( F(\psi, q_t) \) a non-DSGE (time series) component.

(1)-(2) is an extended state space system.

Jointly estimate \((\theta, \psi, \lambda), (\Sigma_e, \Sigma_u, \Sigma_q)\) with standard techniques.

Can use size of \( \text{var}(F(\psi, q_t))/\text{var}(w_t) \) as model fit/comparison device across models with different DSGE features.
Advantages

- Core DSGE structure: can use standard narrative.

- Flexible overcoat: can be adjusted to the needs.

- Use as much information as available for parameter estimation; can examine dynamics to shocks, historical decompositions for variables not in explicitly modelled in the DSGE ($w_{2t}$).

- Inference about latent variables (potential output, natural rates, etc.) uses model structure and all available information.
Potential disadvantages

- Overparameterization.

- Numerical difficulties (nasty objective functions, underidentification of parameters).

- Not distinguishing core vs. non-core for policy analysis/narratives. In a successful model Core structure should explain large portion of observables fluctuations.
What is in $w_t$?

- Standard macro variables.
- Proxies (IP for GDP), flash estimates of standard variables ($\pi_t, \Delta Y_t$).
- Multiple indicators of model variables (e.g. JPT, 2013, AEJM).
- Sectoral data, foreign data, conjunctural data (Gelfer, 2019, RED).
- Nowcasts, forecasts, news, expectation, sentiment and survey data, uncertainty measures.
• Micro data: intermediate step before HANK.

• Can use data at different frequencies (e.g. Foroni and Marcellino, 2014, JOE).

What is $F(\psi, q_t)$?

• Missing idiosyncratic trends (Canova, 2014, JME).

• Missing dynamics.
Proxies

• Let $z^j_t$ be hours and $z^j_t'$ be GDP. Only $w^j_{1t}$ employment and $w^j_{1t}'$ industrial production are available.

$$w^j_{1t} = z^j_t + u^j_t$$
$$w^j_{1t}' = z^j_t' + u^j_t'$$

(3)

Flash estimates

• Let $z^k_t$ be output growth and $w^k_{2t}$ be l=1,2,3...month(s) ahead flash estimates of quarterly output growth.

$$w^k_{2t} = z^k_{t+l} + u^k_t$$

(4)
Multiple indicators

- Let $z^i_t =$ price inflation; $w^i_{3t} =$ CPI inflation, PPI inflation, GDP deflator inflation, etc.

- Measurement equations:

$$
\begin{bmatrix}
    w^1_{3t} \\
    w^2_{3t} \\
    \vdots \\
    w^N_{3t}
\end{bmatrix}
= \begin{bmatrix}
    1 \\
    \lambda_2 \\
    \vdots \\
    \lambda^N
\end{bmatrix} z^i_t + u_t
$$

- $u_t$ vector could be serially correlated as long as $N$ is large. Could restrict the cross sectional variance of $u_t$.

- $\lambda_j$ interpreted as informational content of $w^j_{3t}$ relative to $w^1_{3t}$ (with $\lambda_1$ normalized to 1).
All other cases

• $w_{4t}$ = housing, labor market, financial market variables; sectorial or foreign variables, commodity prices, forecasts, nowcasts, surveys, forward looking indicators (US: Light vehicle sales; price of tomatoes in Munich); quintiles of income/wealth distribution; Fred database.

Measurement equations:

$$w_{4t} = \Lambda x x_t + u_t$$  \hspace{1cm} (6)

$$= \Lambda z z_t + u_t$$  \hspace{1cm} (7)

• $\Lambda$ is a $M \times k$ matrix. Need careful choices of what is in $w_{4t}$ to avoid overparameterization.

• Need $u_t$ to be cross sectionally uncorrelated.
Mixed Frequencies

- Let the DSGE be specified at monthly data, but have $w_{1t}$ at monthly data and $w_{2t}$ at quarterly data.

- Measurement equations (average data):

$$
\begin{bmatrix}
w_{1t} \\
w_{2t}
\end{bmatrix} = \begin{bmatrix}
1/3(z_{2t}^m + z_{2t-1}^m + z_{2t-2}^m)
\end{bmatrix} + \begin{bmatrix}
u_{1t} \\
u_{2t}
\end{bmatrix}
$$

(8)

or (point in time data):

$$
\begin{bmatrix}
w_{1t} \\
w_{2t}
\end{bmatrix} = \begin{bmatrix}
z_{1t}^q \\
z_{2t}^q
\end{bmatrix} + \begin{bmatrix}
u_{1t} \\
u_{2t}
\end{bmatrix}
$$

(9)

some $\bar{t}$. 
Missing trends

- Models typically have balanced growth, e.g., unit root in TFP or in investment specific disturbances or both. Data does not.

• If disregard missing trends or use filtered data, parameter estimates twisted (see Canova, 2014). Parametrize $F(\psi, q_t)$ ('The bridge'), to capture missing features e.g.

\begin{align*}
q_t &= \alpha_t + \rho_1 q_{t-1} + w_{1t} \\
\alpha_t &= \rho_2 \alpha_{t-1} + w_{2t}
\end{align*} \tag{10}

where $w_{2t} \sim (0, \Sigma_{w2})$, $\Sigma_{w2} = \kappa \Sigma_{w1}$.

• $1 + \kappa$ related to $\lambda_{HP}$.

• Up to 2 unit roots or deterministic trends trends are possible. Can be used for selected variables.

• Possibility to generate missing peaks at certain frequencies if $\rho_1, \rho_2 \neq 1$.

• $F(\psi, q_t)$ corresponds to $1 - R^2$. If large, model is poor.
Example: Gelfer (2019)

- Data-rich estimation and forecasting with SWFF. Add 40 sectoral variables in estimation.

- Posterior distributions.

- Impulse responses.

- Estimates of the state variables.

- Forecasts.
Illustration

- Use basic SW model to analyze drivers of the output gap. Four estimated versions:

- Model quarterly; quarterly data on (\(\Delta Y, \Delta C, \Delta I, N, \Delta W, CPIINF, FFR\))

Measurement equations:

\[
\begin{align*}
\Delta Y_t &= 400 * (y_t - y_{t-1}) + c_1 \\
\Delta C_t &= 400 * (c_t - c_{t-1}) + c_2 \\
\Delta I_t &= 400 * (i_t - i_{t-1}) + c_3 \\
H_t &= h_t + c_4 \\
\Delta W_t &= 400 * (w_t - w_{t-1}) + c_5 \\
\Delta P_t &= inft + c_6 \\
FFR_t &= r + c_7
\end{align*}
\]

Note \(c_2 \neq c_2 \neq c_3 \neq c_5\).
• Model monthly, mixed frequency data, \( CPIINF, FFR \) monthly.

Measurement equations:

\[
\begin{align*}
\Delta Y_t &= 400 \times (y_t - y_{t-3}) + c_1 \\
\Delta C_t &= 400 \times (c_t - c_{t-3}) + c_2 \\
\Delta I_t &= 400 \times (i_t - i_{t-3}) + c_3 \\
H_t &= (h_t + h_{t-1} + h_{t-2}) + c_4 \\
\Delta W_t &= 400 \times (w_t - w_{t-3}) + c_5 \\
\Delta P_t &= \inf_t + c_6 \\
FFR_t &= r_t + c_7
\end{align*}
\]
• Model monthly, mixed frequency of data and bridge in FFR.

• Why bridge in FFR?
- Measurements equations: (19)-(24) the same. (25) substituted with

\[
FFR_t = Nr_t + r_t + c_7 \quad (26)
\]
\[
Nr_t = Nr_{t-1} + wr_{t-1} \quad (27)
\]
\[
wr_t = \rho wr_{t-1} + w_t \quad (28)
\]
- Model monthly, mixed frequency, bridge in FFR, other data.

- Measurement equations: (19)-(24)-(26)-(28) plus

\[
\Delta GDPNOW_t = 400 \ast (y_{t+3} - y_t) + u_{1t} \quad (29)
\]
\[
infc_t = infc_{t+12} + u_{2t} \quad (30)
\]
\[
R^t_1 = 1/12(R_t + R_{t+1} + \ldots + R_{t+11}) + c_7 + u_{3t} \quad (31)
\]
- Quarterly model, quarterly data
• Monthly model; mixed frequency data
• Monthly model, mixed frequency data, FFR bridge.
- Monthly model, mixed frequency data, FFR bridge and extra info.
Conclusions

- Considerable uncertainty about the time series features of the output gap and its drivers, even with the same observables. Estimated gap different; driving forces different.

- Model 3 (Mixed frequency data, bridge) more strongly supported by the data using ML comparisons.

- Adding information make estimates of gaps in last 10 years less severe.
References:


