Inflation Persistence, Noisy Information and the Phillips Curve

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*The views expressed do not necessarily reflect the position of Banco de España
Motivation

- Expectations play a central role in (macro)economics

- Most of work considers a limited theory of expectation formation
  - agents are perfectly and homogeneously aware of state and others’ actions

- Explore the nature of expectation formation
  - consistent with data
  - how this matters for macro aggregates and monetary policy

- Significant heterogeneity and sluggishness in inflation expectations
What I do

- Surveys on inflation expectations: Bai-Perron test (structural break)
  - evidence of forecast underreaction before the mid-1980s,
  - not afterward

- Coincides with a change in Fed’s communication strategy, which became more transparent

- Build a New Keynesian model extended with information frictions

- A change in US firms’ belief formation in the mid-1980s can explain two empirical challenges
  - fall in inflation persistence and dynamics of the Phillips Curve
What I find

- Firms’ forecasts used to underreact to information before 1985, not afterwards
  * underreaction: positive co-movement between forecast errors and revisions

\[
\text{forecast error}_t = \pi_{t+4,t} - \hat{F}_t\pi_{t+4,t} \\
\text{revision}_t = \hat{F}_t\pi_{t+4,t} - \hat{F}_{t-1}\pi_{t+4,t}
\]

- Explain the fall in inflation persistence in a NK context
  * inflation is more persistent in periods of forecast stickiness
  * additional persistence in expectations, increasing inflation persistence

- Explain dynamics of the Phillips curve: modest flattening
  * info frictions, Phillips curve is enlarged with anchoring and myopia; changes in backward-lookingness
  * general info structure: only modest flattening once I control for imperfect expectations
Inflation Persistence: the First Puzzle

- Monetary literature documents changes in inflation dynamics over time
- Level, persistence, volatility, ...
- Persistence:
  - Scatter Plot
  - Structural Break
  - Unit Root

  ✱ fall in inflation persistence from 0.75 to 0.5 around 1980-1985 [Fuhrer & Moore (1995), Benati & Surico (2008), Cogley & Sbordone (2008), Cogley, Primiceri & Sargent (2010), Fuhrer (2010), Goldstein & Gorodnichenko (2019)]
  ✱ hard to square in theoretical framework
    + structural shock persistence: stable (monetary, TFP, cost-push)
    + optimal monetary policy: insufficient or unlikely
    + change in trend inflation: insufficient

 Literature Review

# 5
Inflation Persistence: the First Puzzle

- Monetary literature documents changes in inflation dynamics over time
- Level, persistence, volatility,...
- Persistence:
  - Fall in inflation persistence from 0.75 to 0.5 around 1980-1985 [Fuhrer & Moore (1995), Benati & Surico (2008), Cogley & Sbordone (2008), Cogley, Primiceri & Sargent (2010), Fuhrer (2010), Goldstein & Gorodnichenko (2019)]
  - Hard to square in theoretical framework
    - Structural shock persistence: stable (monetary, TFP, cost-push)
    - Optimal monetary policy: insufficient or unlikely
    - Change in trend inflation: insufficient
  - Potential explanations:
    - Cogley et al. (2010): subsample, TR inflation coefficient increased, cost-push shocks less persistent, disturbances less volatile
    - Davig and Doh (2014): regime-switching, TR inflation coefficient increased, fall in volatility of cost-push shocks, explain 40% of fall
    - Bianchi & Ilut (2017): fiscal imbalances and accommodative monpol increase persistence
    - Erceg & Levin (2003): noisy information about CB inflation target explain high persistence in the 1970-80s
  - Contribution: explain this fall through changes in expectations
Exercise 1: study inflation persistence from structural equation, Phillips curve

* Noisy-info Phillips curve

\[ \pi_t = \omega_1 \pi_{t-1} + \kappa \tilde{y}_t + \omega_2 \beta E_t \pi_{t+1} \]

* Evidence of fall in intrinsic persistence \( \omega_1 \to 0 \) and myopia \( \omega_2 \to 1 \)
Exercise 1: study inflation persistence from structural equation, Phillips curve

- Noisy-info Phillips curve
  \[ \pi_t = \omega_1 \pi_{t-1} + \kappa \tilde{y}_t + \omega_2 \beta E_t \pi_{t+1} \]

- Evidence of fall in intrinsic persistence \( \omega_1 \rightarrow 0 \) and myopia \( \omega_2 \rightarrow 1 \)

Exercise 2: slope of Phillips curve controlling for beliefs

- Literature arguing flattening of Phillips Curve
- Inflation no longer affected by demand side (including interest rate)
- In the benchmark NK inflation path given by Phillips curve
  \[ \pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1} \]

- Only possible way: \( \downarrow \kappa \)
- Show that \( \kappa \) has fallen only modestly, and dynamics explained via changes in expectations
Evidence on Imperfect Expectations
Fed Communication History

Since the late 1960s, Fed’s public disclosure and transparency improved

- 1966: FOMC announced decisions once a year (Annual Report)
- 1967: released Policy Report (PR) 90 days after decision
- 1976: PR enlarged and delay reduced to 45 days
- 1976-1993: information contained in PR increased
  - Fed objectives: max employment, stable prices and moderate interest rates
  - macroeconomic forecasts on real GNP and inflation from FOMC members
  - “tilt” (predisposition regarding possible future action)
  - “ranking of policy factors”
  - minutes
- 1994: immediate release of PR after meeting if change
- 1999: immediate release of “tilt”
- 2000: immediate announcement and press conference after meeting
Survey of Professional Forecasters, 1968:Q4-2020:Q1

- conducted by ASA, NBER and Philly Fed
- every quarter forecasters asked on forecasts of macro variables
- asked to give nowcast, quarter-ahead forecast, etc. up to five quarters
- forecasters work at Wall Street financial firms, commercial banks, consulting firms, research centers and other private sector companies
- used extensively in the literature [Coibion & Gorodnichenko (2012, 2015), Bordalo et al. (2020), Broer & Kohlhas (2021)]

Results robust to Livingston Survey
Evidence on expectations

- Focus on annual inflation forecasts forecasting frictions

- Coibion & Gorodnichenko (2012, 2015): positive co-movement between ex-ante forecast error and forecast revision

$$\text{forecast error}_t = \pi_{t+4} - F_t \pi_{t+4}, \quad \text{revision}_t = F_t \pi_{t+4} - F_{t-1} \pi_{t+4}$$
Evidence on expectations

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\]
Underrevision behavior pre-1985...

**Figure** First-Vintage inflation and forecasts

![Graph showing the relationship between forecast revision and forecast error for the years 1969-1984. The graph includes a linear fit line for the data from 1969-1984.](image-url)
Underrevision behavior vanished!

**Figure** First-Vintage inflation and forecasts
\[
\text{forecast error}_t = \alpha_{\text{rev}} + \beta_{\text{rev}} \text{revision}_t + \beta_{\text{rev},*} \text{revision}_t \times 1_{\{t \geq t^*\}} + u_t
\]

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Robust standard errors in parenthesis
Control: constant
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Additional Evidence: Forecast Errors and Monetary Shocks

- Estimate IRFs of forecast error on Romer & Romer monetary shocks

\[
\text{forecast error}_{t+h} = \beta_h \epsilon_t + \gamma X_t + u_t
\]

\* \( X_t = \{4 \text{ lags of R&R shocks, 4 lags of FE}\) 

- Test for a change after 1985

\[
\text{forecast error}_{t+h} = (\beta_h + \beta_h^* \times 1_{t \geq t^*}) \epsilon_t + \gamma X_t + u_t
\]

- Results consistent with a fall in information frictions
Additional Evidence: Forecast Errors and Monetary Shocks

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\( X_t = \{4 \text{ lags of R&R shocks, } 4 \text{ lags of FE}\)  

- Test for a change after 1985

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\text{forecast error}_{t+h} = (\beta_h + \beta_h^* \times 1_{t \geq t^*}) \varepsilon_t + \gamma X_t + u_t
\]

- Results consistent with a fall in information frictions
Additional Evidence: Disagreement and Monetary Shocks

- **disagreement** at time $t$: cross-sectional standard deviation of forecasts at time $t$

  $$\text{disagreement}_t = \sigma_i(F_{it} \pi_{t+4,t})$$

- Estimate IRFs of forecast error on Romer & Romer monetary shocks, test for change after 1985

  $$\text{disagreement}_{t+h} = (\beta_h + \beta_h^* \times \mathbb{1}_{\{t \geq t^*\}}) \epsilon_t + \gamma X_t + u_t$$

- **Sticky information**: disagreement should increase after a monetary shock
- **Noisy information**: disagreement should not react to monetary shocks
- **Full information**: no reaction
Additional Evidence: Disagreement and Monetary Shocks

- \textit{disagreement} at time $t$: cross-sectional standard deviation of forecasts at time $t$

  \[ \text{disagreement}_t = \sigma_i(F_{it} \pi_{t+4, t}) \]

- Estimate IRFs of forecast error on Romer & Romer monetary shocks, test for change after 1985

  \[ \text{disagreement}_{t+h} = (\beta_h + \beta_h * \mathbb{1}_{\{t \geq t^*\}}) \varepsilon_t + \gamma X_t + u_t \]

  - \textit{Sticky information}: disagreement should increase after a monetary shock
  - \textit{Noisy information}: disagreement should not react to monetary shocks
  - \textit{Full information}: no reaction

\textbf{Evidence on Imperfect Expectations Theory Results Conclusion}
Survey conducted semiannually, estimate the following structural-break variant:

\[ \pi_{t+2} - F_t \pi_{t+2} = \alpha_{rev} + \beta_{rev} (F_t \pi_{t+2} - F_{t-2} \pi_{t+2}) + u_t \]

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<td>CG Regression</td>
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<tr>
<td>Revision</td>
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<td>0.412** (0.204)</td>
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<tr>
<td>Revision ( \times ^{\mathbb{1}_{t\geq t^*}} )</td>
<td></td>
<td>-0.880** (0.414)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.183* (0.102)</td>
<td>-0.105 (0.119)</td>
</tr>
<tr>
<td>Observations</td>
<td>146</td>
<td>146</td>
</tr>
</tbody>
</table>

HAC robust standard errors in parentheses

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Model in a nutshell: NK + Noisy & Dispersed Information

- Theory consistent with underreaction

- New Keynesian + noisy information
  - househols and central bank NK-standard
  - firms are subject to information frictions
  - signal extraction problem: observe imprecise signal on monetary shock

- Endogenous forecast underreaction: shrink forecast towards prior beliefs

- Translates into inflation persistence
Consumers are NK-standard

- Continuum of infinitely-lived, ex-ante identical households

- Consume a CES bundle of $j \in [0, 1]$ goods with elasticity $\varepsilon$

- Cost-minimization: demand function $c_{jt} = \left( \frac{p_{jt}}{p_t} \right)^{-\varepsilon} c_t$ and price index $p_t \equiv \left( \int p_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$

- Households maximize $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ subject to budget constraint
  
  $C_t + B_t = R_{t-1} B_{t-1} + \frac{W_t}{P_t} N_t + T_t$
Consumers are NK-standard

- Continuum of infinitely-lived, ex-ante identical households
- Consume a CES bundle of \( j \in [0, 1] \) goods with elasticity \( \epsilon \)
- Cost-minimization: demand function \( C_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\epsilon} C_t \) and price index \( P_t \equiv \left( \int P_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \)
- Households maximize \( E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \) subject to budget constraint
  \[ C_t + B_t = R_{t-1} B_{t-1} + \frac{W_t}{P_t} N_t + T_t \]
- Optimality conditions under CRRA preferences
  \[ c_t = -\frac{1}{\sigma} E_t (i_t - \pi_{t+1}) + E_t c_{t+1}, \quad w_t - p_t = \sigma c_t + \varphi n_t \]
Central Bank is NK-standard

- Central bank sets nominal interest rates following a Taylor rule

\[ i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \]

- Reacts to excess inflation \( \pi_t \) and output gap \( \tilde{y}_t = y_t - y^n_t \)
Central Bank is NK-standard

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- Reacts to excess inflation \( \pi_t \) and output gap \( \tilde{y}_t = y_t - y^n_t \)

- Monetary shock \( v_t \) follows an AR(1) process
  \[ v_t = \rho v_{t-1} + \sigma \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1) \]

- Key object: firms observe an imprecise signal of \( v_t \)
Firms: Island Model

- Know island conditions (prices, production)
- Imprecise idea of the aggregate archipelago conditions
Firms: Island Model

- Know island conditions (prices, production)
- Imprecise idea of the aggregate archipelago conditions
- Continuum of firms producing a differentiated intermediate good variety $j$
- Set price $P_{jt}$ and face demand $Y_{jt}$
- Use technology $Y_{jt} = N_{jt}^{1-\alpha}$
- Nominal price rigidity: Calvo-lottery friction (at every period, each firm is able to reset price with probability $1 - \theta$)

Evidence on Imperfect Expectations Theory Results Conclusion
Share of Re-setters, $1 - \theta$

- Set price $P_{jt}^*$ to maximize (real) profits while price remains effective

$$P_{jt}^* = \arg \max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k E_{jt} \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[ P_{jt} Y_{j,t+k|t}(P_{jt}) - W_{t+k} N_{j,t+k|t}(P_{jt}) \right] \right\}$$

s.t. $Y_{j,t+k|t} = \left( \frac{P_{jt}}{P_{t+k}} \right)^{-\epsilon} C_{t+k}, \quad Y_{j,t+k|t} = N_{1-\alpha}^{j,t+k|t}$

- $E_{jt}(\cdot) :=$ firm $j$’s expectation conditional on its information set at time $t$, stochastic discount factor $\Lambda_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma}$
Share of Re-setters, $1 - \theta$

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s.t. $Y_{j,t+k|t} = \left( \frac{P_{jt}}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$, $Y_{j,t+k|t} = N_{1-\alpha}$

- $\mathbb{E}_{jt}(\cdot) :=$ firm $j$’s expectation conditional on its information set at time $t$, stochastic discount factor $\Lambda_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma}$

- Recursive price-setting condition

$$p_{jt}^* = (1 - \beta \theta) \mathbb{E}_{jt} p_t + \frac{\kappa \theta}{1 - \theta} \mathbb{E}_{jt} \tilde{y}_t + \beta \theta \mathbb{E}_{jt} p_{j,t+1}^*$$

$$\kappa \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon}$$

- Firm $j$ needs to infer others’ decisions: $p_t = (1 - \theta) \int_{I_f} p_{jt}^* \, dj + \theta p_{t-1} = (1 - \theta) \sum_{k=0}^{\infty} \theta^k p_{t-k}^*$
Information Structure

- Each firm $j$ observes noisy signal $x_{jt}$ on the monetary shock $v_t$,

$$
x_{jt} = v_t + \sigma_u u_{jt}, \quad \text{with } u_{jt} \sim \mathcal{N}(0, 1)
$$

- Information on state $v_t$, aggregate demand $\tilde{y}_t(v_t)$ and others’ actions $p_t(v_t)$

- Information is imprecise, firms do not fully react to $x_{jt}$
Information Structure

▶ Each firm $j$ observes noisy signal $x_{jt}$ on the monetary shock $v_t$.

\[ x_{jt} = v_t + \sigma_u u_{jt}, \quad \text{with } u_{jt} \sim \mathcal{N}(0, 1) \]

▶ Information on state $v_t$, aggregate demand $\tilde{y}_t(v_t)$ and others’ actions $p_t(v_t)$

▶ Information is imprecise, firms do not fully react to $x_{jt}$

▶ NK framework linear: bayesian updating (Kalman/Wiener-Hopf filter)

\[ \mathbb{E}_{jt} z_t = \Lambda(\sigma_u) \mathbb{E}_{j, t-1} z_{t-1} + K(\sigma_u) x_{jt}, \quad z_t = \begin{bmatrix} v_t & p_t & \tilde{y}_t \end{bmatrix}^T \]

▶ Forecasts react sluggishly if $\Lambda \neq 0$

▶ Noisy information generates additional persistence
Recap: In equilibrium...

- DIS curve

\[
\tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + E_t \tilde{y}_{t+1}
\]

- Taylor rule

\[
i_t = \phi_{\pi} \pi_t + \phi_y \tilde{y}_t + \nu_t, \quad \nu_t = \rho \nu_{t-1} + \sigma_\varepsilon \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1)
\]

- Individual price-setting:

\[
p^*_jt = (1 - \beta \theta) E_{jt} p_t + \frac{\kappa \theta}{1 - \theta} E_{jt} \tilde{y}_t + \beta \theta E_{jt} p^*_{j,t+1}
\]

\[
E_{jt} z_t = \Lambda E_{j,t-1} z_{t-1} + Kx_{jt}, \quad x_{jt} = \nu_t + \sigma_u u_{jt}, \quad \text{with } u_{jt} \sim \mathcal{N}(0, 1)
\]
Inflation Dynamics

Benchmark \( (\sigma_u = 0) \)

- **Reduced-form:**
  \[
  \pi_t = \psi_{\pi} v_t \\
  \psi_{\pi} = \frac{-\kappa \sigma_\epsilon}{(1 - \rho \beta)(\sigma(1 - \rho) + \phi_y) + \kappa(\phi_{\pi} - \rho)}
  \]

- **Structural-form:**
  \[
  \pi_t = \kappa \tilde{y}_t + \beta \bar{E}_t \pi_{t+1}
  \]
Inflation Dynamics

Benchmark \((\sigma_u = 0)\)

▶ Reduced-form: Derivation

\[ \pi_t = \psi_{\pi} v_t \]

\[ \psi_{\pi} = \frac{-\kappa \sigma_{\epsilon}}{(1 - \rho \beta)[\sigma(1 - \rho) + \phi_y] + \kappa(\phi_{\pi} - \rho)} \]

▶ Structural-form:

\[ \pi_t = \kappa \tilde{y}_t + \beta \bar{E}_t \pi_{t+1} \]

Noisy Information

▶ Reduced-form: Proposition

\[ \pi_t = \delta \pi_{t-1} + \xi \pi_{t-2} + \psi_{\pi} \chi \Delta v_t \]

\[ \delta(\sigma_u, \Phi), \xi(\sigma_u, \Phi) \text{ and } \chi(\sigma_u, \Phi) \text{ are scalars} \]

endogenous to information frictions \(\sigma_u\)

▶ Structural-form: Proposition

\[ \pi_t = \omega_1 \pi_{t-1} + \kappa \tilde{y}_t + \omega_2 \beta \bar{E}_t \pi_{t+1} \]

wedge Phillips curve produces identical dynamics for certain values of \((\omega_1, \omega_2) \in [0, 1]^2\)

More on \(\delta, \xi, \text{ and } \chi\)
Results
Policy Experiments

How does the change in information frictions ($\sigma_u$)/sluggishness ($\beta_{rev}$) affect (1) inflation persistence, and (2) the dynamics of the Phillips curve?
Policy Experiments

How does the change in information frictions ($\sigma_u$)/sluggishness ($\beta_{rev}$) affect (1) inflation persistence, and (2) the dynamics of the Phillips curve?

Table Model parameters.

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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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Results: Persistence
First Order Autocorrelation

- Inflation first-order autocorrelation $\rho_1$

$$\rho_1 = \frac{1}{2} \frac{(1 + \rho) \delta - (1 - \rho)(1 + \xi)}{1 - \rho \xi}$$
First Order Autocorrelation

- Inflation first-order autocorrelation $\rho_1$

\[
\rho_1 = \frac{1}{2} \frac{(1 + \rho)\delta - (1 - \rho)(1 + \xi)}{1 - \rho \xi}
\]

- Increasing in $\sigma_u$

**Figure** First-order autocorrelation $\rho_1$ and information frictions $\tau^{-1} = \sigma_u^2 / \sigma_\epsilon^2$
Information Frictions Regression

▶ Forecast underrevision

\[ \beta_{rev} = \frac{\mathbb{C}(\text{forecast error}_t, \text{revision}_t)}{\mathbb{V}(\text{revision}_t)} = f(\sigma_u, \Phi) \]

▶ Increasing in \( \sigma_u \) Proposition

Figure \( \beta_{rev} \) and information frictions \( \tau^{-1} = \sigma_u^2 / \sigma_\epsilon^2 \)
First-order Autocorrelation and Underrevision

- Inflation first-order autocorrelation $\rho_1(\sigma_u, \Phi)$ is increasing in $\beta_{rev}$

**Figure** Autocorrelation $\rho_1$ and $\beta_{rev}$
Expectations can explain inflation persistence fall!

- Calibrate signal noise $\sigma_u$ to match empirical evidence on $\beta_{\text{rev}}(\sigma_u, \Phi)$

Pre-1985:

\[
\beta_{\text{rev}}(\sigma_u, \Phi) = 1.501 \\
\sigma_u = 2.501
\]
Expectations can explain inflation persistence fall!

Calibrate signal noise $\sigma_u$ to match empirical evidence on $\beta_{\text{rev}}(\sigma_u, \Phi)$

Pre-1985:

$$\beta_{\text{rev}}(\sigma_u, \Phi) = 1.501$$
$$\sigma_u = 2.501$$

Post 1985:

$$\beta_{\text{rev}}(\sigma_u, \Phi) = 0$$
$$\sigma_u = 0$$
Expectations can explain inflation persistence fall!

Calibrate signal noise $\sigma_u$ to match empirical evidence on $\beta_{\text{rev}}(\sigma_u, \Phi)$

Pre-1985:

$\beta_{\text{rev}}(\sigma_u, \Phi) = 1.501$

$\sigma_u = 2.501$

Post 1985:

$\beta_{\text{rev}}(\sigma_u, \Phi) = 0$

$\sigma_u = 0$

Table First Order Autocorrelation $\rho_1$, Data vs. Model

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<td>Model</td>
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<td>0.500</td>
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</table>
Inflation persistence fell since mid 1980s

Hard to understand in NK setting

Document a new empirical result on information frictions

A model consistent with this finding explains around 90% of its fall
Results: Phillips curve
Exercise 1: Change in Phillips curve slope

Noisy information pre-1985: 

\[ \pi_t = \omega_1 \pi_{t-1} + \kappa \tilde{y}_t + \omega_2 \beta E_t \pi_{t+1} \]

* wedge Phillips curve produces identical dynamics for certain values of \((\omega_1, \omega_2) \in [0, 1]^2\)
* \(\omega_1 \in (0, 1)\): anchoring
* \(\omega_2 \in (0, 1)\): myopia

Simulated Data

| Regression | \(\pi_t - 1\) | \(\pi_t - 1 \times 1_{\{t \geq t^*\}}\) | \(\pi_{t+1}\) |  \\
<table>
<thead>
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<td></td>
<td>0.720***</td>
<td>-0.597**</td>
<td>0.273**</td>
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<tr>
<td></td>
<td>(0.131)</td>
<td>(0.232)</td>
<td>(0.129)</td>
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<tr>
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<td>0.0566</td>
<td>-0.0143</td>
<td>0.643***</td>
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<tr>
<td></td>
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<td>(0.0488)</td>
<td>(0.244)</td>
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Observations 202

HAC robust standard errors in parentheses

Instrument set: four lags of effective federal funds rate, CBO Output gap, GDP Deflator growth rate, Commodity inflation, M2 growth rate, spread between long and short-run interest rate and labor share.
Exercise 1: Change in Phillips curve slope

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- \(\omega_1 \in (0, 1)\): anchoring
- \(\omega_2 \in (0, 1)\): myopia

No information frictions post-1985:

\[ \pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1} \]

- \(\omega_1 = 0, \omega_2 = 1\)

Simulated Data
Exercise 1: Change in Phillips curve slope

Noisy information pre-1985: Proposition

\[
\pi_t = \omega_1 \pi_{t-1} + \kappa \tilde{y}_t + \omega_2 \beta E_t \pi_{t+1}
\]

* wedge Phillips curve produces identical dynamics for certain values of \((\omega_1, \omega_2) \in [0, 1]^2\)
  * \(\omega_1 \in (0, 1):\) anchoring
  * \(\omega_2 \in (0, 1):\) myopia

No information frictions post-1985:

\[
\pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1}
\]

* \(\omega_1 = 0, \; \omega_2 = 1\)

Table Regression table

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<td>(0.131)</td>
</tr>
<tr>
<td>(\pi_{t-1} \times 1_{{t \geq t^*}})</td>
<td>-0.597**</td>
</tr>
<tr>
<td></td>
<td>(0.232)</td>
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<tr>
<td>(\tilde{y}_t)</td>
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<tr>
<td></td>
<td>(0.0488)</td>
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<tr>
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<td>(\pi_{t+1})</td>
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<td>(\pi_{t+1} \times 1_{{t \geq t^*}})</td>
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<td></td>
<td>(0.244)</td>
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Observations 202

HAC robust standard errors in parentheses

Instrument set: four lags of effective federal funds rate, CBO Output gap, GDP Deflator growth rate, Commodity inflation, M2 growth rate, spread between long and short-run interest rate and labor share.

Evidence on Imperfect Expectations Theory Results Conclusion # 29
Exercise 2: Imperfect Expectations

- Agnostic stance on belief formation
- Aggregate Phillips curve

\[ \pi_t = \kappa \theta \sum_{k=0}^{\infty} (\beta \theta)^k \bar{E}_t \bar{y}_{t+k} + (1 - \theta) \sum_{k=0}^{\infty} (\beta \theta)^k \bar{E}_t \pi_{t+k} + \left( \bar{E}_t p_{t-1} - p_{t-1} \right) \]

- Average firm’s expectation \( \bar{E}_t (\cdot) = \int E_{jt}(\cdot) \, dj \)
- Have data on \( \bar{E}_t \bar{y}_{t+k} \) and \( \bar{E}_t \pi_{t+k} \)!
- Test for a break in \( \kappa \) controlling for imperfect expectations
- Set \( \beta = 0.99 \) and \( \theta = 0.89 \), truncate sums at \( k = 4 \):

\[ \pi_t = \kappa y_t^e + (1 - \theta) \pi_t^e + \eta_t, \quad \eta_t = \left( \bar{E}^f_t p_{t-1} - p_{t-1} \right) + \text{truncation error} \]
### Table: Estimates of regression.

<table>
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<tr>
<th></th>
<th>Unemployment</th>
<th>Real GDP Growth</th>
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<tr>
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<td>Full Sample</td>
<td>Structural Break</td>
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<td>( \tilde{y}_t^e )</td>
<td>-0.00519***</td>
<td>-0.0231***</td>
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<tr>
<td></td>
<td>(0.00171)</td>
<td>(0.00679)</td>
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<tr>
<td>( \tilde{y}<em>t^e \times \mathbb{1}</em>{{t \geq t^* }} )</td>
<td>0.0133***</td>
<td>-0.0403**</td>
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<td>(0.00493)</td>
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<tr>
<td>( \pi_t^e )</td>
<td>0.282***</td>
<td>0.342***</td>
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<td>(0.0109)</td>
<td>(0.0261)</td>
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<td>Observations</td>
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<td>199</td>
</tr>
</tbody>
</table>

HAC (1 lag) robust standard errors in parentheses. Instrument set: four lags of forecasts of annual real GDP growth and annual GDP Deflator growth.

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

- Modest fall in \( \kappa \), consistent with Hazell et al. (2022)
Conclusion
A change in US firms’ belief formation in the mid-1980s can explain two empirical challenges:

- fall in inflation persistence
- flattening of the Phillips curve

Explain around 90% of fall in inflation persistence through changes in expectations:

- Inflation is forward-looking, endogenous to expectations
- Forecast underreaction generates persistence in expectations

Explain changing dynamics in Phillips curve through changes in expectations:

- Re-shuffle between backward- and forward-lookingness
- Modest flattening after controlling for imperfect expectations

Will the 2020-22 inflation be persistent?

Fed should pay attention to forecast underrevision!
Conclusion

- A change in US firms’ belief formation in the mid-1980s can explain two empirical challenges
  - fall in inflation persistence
  - flattening of the Phillips curve

- Document forecast underreaction before mid 1980s, not afterwards
  - positive co-movement between forecast errors and revisions
A change in US firms’ belief formation in the mid-1980s can explain two empirical challenges

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Evidence on Imperfect Expectations Theory Results Conclusion
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Will the 2020-22 inflation be persistent? Fed should pay attention to forecast underrevision!
Will the 2020-22 inflation be persistent? (speculative) yes...
Will the 2020-22 inflation be persistent? (speculative) yes...

Evidence on Imperfect Expectations Theory Results Conclusion

# 33
Thank you!
Structural Break Test
### Table Structural break

<table>
<thead>
<tr>
<th>Error vs. Revision</th>
<th>F-Statistic</th>
<th>p-value</th>
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</thead>
<tbody>
<tr>
<td>1980:Q3</td>
<td>11.25</td>
<td>0.00</td>
</tr>
<tr>
<td>1985:Q1</td>
<td>7.96</td>
<td>0.01</td>
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</table>

<table>
<thead>
<tr>
<th>Inflation Persistence</th>
<th>F-Statistic</th>
<th>p-value</th>
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</thead>
<tbody>
<tr>
<td>1991:Q1</td>
<td>32.03</td>
<td>0.00</td>
</tr>
<tr>
<td>1985:Q1</td>
<td>28.22</td>
<td>0.00</td>
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Benchmark NK
**Benchmark**

- **Dynamic IS curve**
  \[
  \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + E_t \tilde{y}_{t+1}
  \]  (1)

- **NK Phillips curve**
  \[
  \pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1}
  \]  (2)

- **Monetary policy rule**
  \[
  i_t = \phi_\pi \pi_t + \phi_y y_t + v_t,
  \quad
  v_t = \rho v_{t-1} + \epsilon_t^v,
  \quad
  \epsilon_t^v \sim N(0, \sigma_\epsilon^2)
  \]  (3)

- Introducing (3) into (1), we can write (1)-(2) as a system of two first-order forward-looking stochastic equations
- Inflation dynamics are given by
  \[
  \pi_t = -\psi_\pi v_t
  \]
  \[
  = \rho \pi_{t-1} - \psi_\pi \epsilon_t
  \]
Measuring the Shock Process

Problem: $v_t$ is unobservable, but we have estimates on monetary policy shocks $\varepsilon_t^v$ from Romer and Romer (2004), updated until 2007 by Wieland & Yang (2020).

Solution: indirect estimation on $\rho$

Using the AR(1) property of the $v_t$ shock process, we can write the Taylor rule as

$$i_t = \rho i_{t-1} + \left( \phi_\pi \pi_t + \phi_y y_t \right) - \rho \left( \phi_\pi \pi_{t-1} + \phi_y y_{t-1} \right) + \varepsilon_t^v \quad (4)$$

An estimate of the first autoregressive coefficient identifies monetary policy persistence.
Persistence

\[ i_t = \rho i_{t-1} + \left( \phi_{\pi} \pi_t + \phi_y y_t \right) - \rho \left( \phi_{\pi} \pi_{t-1} + \phi_y y_{t-1} \right) + \epsilon_t^v \]  

(5)

- Structural break analysis
- Estimate using unrestricted GMM

\[ i_t = \alpha_i + \alpha_{i, \ast} \mathbb{I}_{\{t \geq t^*\}} + \rho_i i_{t-1} + \rho_{i, \ast} i_{t-1} \mathbb{I}_{\{t \geq t^*\}} + \gamma X_{t, t-1} + u_t \]

- Notice: \( \rho \) also interacts with lagged inflation and output gap in (5)
- Estimate structural break in (5), restricted GMM
\[ i_t = \alpha_i + \alpha_{i, \ast} I_{\{t \geq t^* \}} + \rho_i i_{t-1} + \rho_{i, \ast} i_{t-1} I_{\{t \geq t^* \}} + \gamma X_{t-1} + \epsilon_t \]

<table>
<thead>
<tr>
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<th>(1) Unrestricted GMM</th>
<th>(2) GMM</th>
<th>(3) Restricted GMM</th>
<th>(4) Restricted GMM</th>
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<tr>
<td>(i_{t-1})</td>
<td>0.941*** (0.0184)</td>
<td>0.939*** (0.0448)</td>
<td>0.972*** (0.0119)</td>
<td>0.931*** (0.0365)</td>
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<tr>
<td>(i_{t-1} \times I_{{t \geq t^* }})</td>
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<td></td>
<td>-0.0537 (0.0632)</td>
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<td>Constant</td>
<td>0.122 (0.118)</td>
<td>0.305 (0.473)</td>
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<td>0.851** (0.373)</td>
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<td>-0.813 (0.559)</td>
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<td>Observations</td>
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<td>203</td>
<td>203</td>
<td>203</td>
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</tbody>
</table>

Standard errors in parentheses
* \(p < 0.10\), ** \(p < 0.05\), *** \(p < 0.01\)

- Benchmark NK model cannot explain the fall in inflation persistence
- Inherited from monetary shock process, did not change
Technology and Cost-push Shocks

- Extend the basic framework to demand (technology) and supply (cost-push) shocks, $a_t$ and $u_t$

- Demand side:

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t \pi_{t+1}) - (1 - \rho_a)\psi_y a_t + E_t \tilde{y}_{t+1}$$

- Supply side:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t + u_t$$

- $a_t$ and $u_t$ follow AR(1) processes with persistence $\rho_a$ and $\rho_u$

- Inflation dynamics follow:

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t$$
First-order autocorrelation coefficient $\rho_1$ depends critically on the $\rho_x$'s

$$\rho_1 = \frac{\rho \frac{\psi_{\pi v}^2 \sigma_{\varepsilon v}^2}{1-\rho_v^2} + \rho_a \frac{\psi_{\pi a}^2 \sigma_{\varepsilon a}^2}{1-\rho_a^2} + \rho_u \frac{\psi_{\pi u}^2 \sigma_{\varepsilon u}^2}{1-\rho_u^2}}{\frac{\psi_{\pi v}^2 \sigma_{\varepsilon v}^2}{1-\rho_v^2} + \frac{\psi_{\pi a}^2 \sigma_{\varepsilon a}^2}{1-\rho_a^2} + \frac{\psi_{\pi u}^2 \sigma_{\varepsilon u}^2}{1-\rho_u^2}}$$

- We already documented no change in $\rho$
- Find evidence on a structural break in $\rho_a$ and $\rho_u$
Technology Shock

- Use three data series used in the literature
- Fernald (2014) estimates directly (log) technology $a_t$
- Francis et al. (2014) and Justiniano et al. (2011) estimate the technology shock $\varepsilon_t^a$
  - Indirect estimation of $\rho_a$ using the natural real interest rate process
  - Natural real rate $r_t^n = -\sigma \psi_y a_t (1 - \rho_a)$
  - Fed estimate of natural rate, produced by Holston (2017)

\[
r_t^n = \rho_a r_{t-1}^n - \sigma \psi_y (1 - \rho_a) \varepsilon_t^a
\]
<table>
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<th></th>
<th>(1) Technology</th>
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<tr>
<td>Natural rate_{t-1}</td>
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</tr>
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</table>

Robust standard errors in parentheses
* p < 0.10, ** p < 0.05, *** p < 0.01
Cost-Push Shock

- Nekarda & Ramey (2010) estimate the structural time-varying price-cost markup
- Two different measures of the cost-push shock
  - Assume Cobb-Douglas production function
  - Assume CES production function, estimating labor-augmented technology using long-run restrictions as in Gali (1999)

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tbody>
<tr>
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<td>0.938***</td>
<td>0.963***</td>
<td>0.947***</td>
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<tr>
<td>(0.0246)</td>
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<td>Constant</td>
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<td>0.0252**</td>
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<td>(0.0125)</td>
<td>(0.0146)</td>
<td>(0.0117)</td>
<td>(0.0120)</td>
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<td>Observations</td>
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</tr>
</tbody>
</table>

Standard errors in parentheses

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Optimal Monetary Policy under Discretion

- Pre-1985, inflation dynamics
  \[\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t\]

- Post-1985 with optimal policy, CB minimizes welfare losses
  \[E_0 \sum_{k=0}^{\infty} \beta^k \left( \pi_t^2 + \frac{K}{\epsilon} x_t^2 \right)\]

  \[x_t \equiv \text{welfare-relevant output gap, subject to Phillips curve}\]
  \[\pi_t = k x_t + \xi_t,\]

  \[\xi_t \equiv \beta E_{t} \pi_{t+1} + u_t \text{ non-policy shock}\]

- Inflation dynamics
  \[\pi_t = \rho_u \pi_{t-1} + \psi_d \xi^u_t\]

- Persistence inherited from cost-push shock
- No significant change in persistence: pre-1985 persistence around 0.95, post around 0.96
Optimal Monetary Policy under Commitment

- Pre-1985 period inflation dynamics

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t$$

- Post-1985 with optimal policy, CB minimizes welfare losses

$$E_0 \sum_{k=0}^{\infty} \beta^t \left( \pi_t^2 + \frac{K}{\varepsilon} x_t^2 \right)$$

$$x_t \equiv \text{welfare-relevant output gap, subject to Phillips curve}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

- Inflation dynamics

$$\pi_t = \rho_c \pi_{t-1} + \psi_c \Delta u_t$$

- $\rho_c$ depends on deep parameters

- Commitment requires change $\phi_\pi$ from 1 to 6.5, inconsistent with empirical evidence
Price Indexation

- Generate intrinsic persistence through price indexation
- Restricted firms reset price indexed to past inflation: $p_{it} = p_{i,t-1} + \omega \pi_{t-1}$
- Phillips curve modified to

$$\Delta_t = \kappa \tilde{y}_t + \beta E_t \Delta_{t+1},$$

where $\Delta_t := \pi_t - \omega \pi_{t-1}$

- Inflation dynamics

$$\pi_t = \rho \omega \pi_{t-1} + \psi \omega \nu_t$$
Trend Inflation

- Ascari & Sbordone (2014), Stock & Watson (2007): fall in trend inflation from 4% to 2%
- Log-linearize around positive trend inflation
- Phillips curve now a system of three equations

\[
\begin{align*}
\pi_t &= \Xi_1 \psi_t + \Xi_2 y_t + \Xi_3 E_t \psi_{t+1} + \Xi_4 E_t \pi_{t+1} \\
\psi_t &= \Gamma_1 s_t + \Gamma_2 y_t + \Gamma_3 E_t \psi_{t+1} + \Gamma_4 E_t \pi_{t+1} \\
s_t &= \Lambda_1 \pi_t + \Lambda_2 s_{t-1}
\end{align*}
\]

- $\Lambda_2(\bar{\pi})$ increasing in $\bar{\pi}$
- Inflation dynamics

\[
\pi_t = \rho_{\bar{\pi}} \pi_{t-1} + \psi_{\bar{\pi}} \nu_t + \xi_t,
\]

where $\xi_t$ MA($\infty$) process and $\rho_{\bar{\pi}}$ increasing in $\bar{\pi}$
Inflation Persistence
Figure Inflation Persistence, 1969-1984
Figure: Inflation Persistence, 1969-1984 and 1985-2020
 structural break

Table \( \pi_t = \alpha_{\pi} + \alpha_{\pi, \ast} \mathbb{1}_{\{t \geq t^*\}} + (\rho_{\pi} + \rho_{\pi, \ast} \mathbb{1}_{\{t \geq t^*\}})\pi_{t-1} + \epsilon_t^{\pi} \)

<table>
<thead>
<tr>
<th></th>
<th>All Sample</th>
<th>Structural Break</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{t-1} )</td>
<td>0.880***</td>
<td>0.785***</td>
</tr>
<tr>
<td></td>
<td>(0.0466)</td>
<td>(0.0755)</td>
</tr>
<tr>
<td>( \pi_{t-1} \times \mathbb{1}_{{t \geq t^*}} )</td>
<td>-0.287**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.144)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.400**</td>
<td>1.320***</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.471)</td>
</tr>
<tr>
<td>Constant ( \times \mathbb{1}_{{t \geq t^*}} )</td>
<td>-0.263</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.543)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>206</td>
<td>206</td>
</tr>
</tbody>
</table>

HAC robust standard errors in parenthesis,
* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Unit Root Test

- Cross-sample unit root analysis
  - Augmented Dickie-Fuller
  - Phillips-Perron
- Null hypothesis (unit root) cannot be rejected in the pre-1985 sample
- Strong rejection of the null in the post-1985 sample

<table>
<thead>
<tr>
<th>p-values, null = series has unit root</th>
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<tbody>
<tr>
<td>1969-2020</td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>GDP Deflator</td>
</tr>
<tr>
<td>CPI</td>
</tr>
<tr>
<td>PCE</td>
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<tr>
<td>1969-1985</td>
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<tr>
<td>Variable</td>
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<tr>
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<tr>
<td>CPI</td>
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<td>1985-2020</td>
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<tr>
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<tr>
<td>GDP Deflator</td>
</tr>
<tr>
<td>CPI</td>
</tr>
<tr>
<td>PCE</td>
</tr>
</tbody>
</table>

Back to "Inflation Persistence: the First Puzzle"
Back to "Expectations can Explain..."
Literature Review on Persistence

- Barsky (1987): historical analysis (1839-1979) documenting time-varying persistence
- Pivetta & Reis (2007): within decade variation in persistence
- Benati (2008): international analysis, inflation targeting reduces inflation persistence
- Fuhrer (2010): inflation persistence fell since 1985
- Goldstein & Gorodnichenko (2020): forecast- *implied* persistence fell gradually since 1968

**Table** First Order Autocorrelation, Inflation (Q-to-Q).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Deflator</td>
<td>0.7572</td>
<td>0.4968</td>
</tr>
<tr>
<td>CPI</td>
<td>0.7856</td>
<td>0.2898</td>
</tr>
<tr>
<td>PCE</td>
<td>0.8047</td>
<td>0.4086</td>
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</tbody>
</table>

Back to "Inflation Persistence: the First Puzzle"
Forecast Underrevision
Interpretation

\[ \text{forecast error}_t = \beta_{\text{rev}} \text{revision}_t + e_t \]

- Econometrician does not know what exactly happened between \( t - 1 \) and \( t \)
- Can observe the forecast revision
- Suppose revision\(_t > 0\)
- \( \beta_{\text{rev}} > 0 \) implies that forecast error\(_t > 0\)
- \( \pi_t + 4 = \mathbb{F}_t \pi_{t+4} > 0 \)
Outliers

Figure First-Vintage inflation and forecasts
Outliers

Figure First-Vintage inflation and forecasts
Outliers

**Figure** First-Vintage inflation and forecasts

Back to "Underrevision Behavior has Vanished"
Table \( \text{forecast error}_t = \alpha + (\beta_{\text{rev}} + \beta_{\text{rev},*} \times 1_{\{t \geq t^*\}}) \text{revision}_t + \epsilon_{\text{rev}}^t \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Revision</td>
<td>1.230***</td>
<td>1.414***</td>
<td>0.169</td>
<td>1.501***</td>
</tr>
<tr>
<td></td>
<td>(0.250)</td>
<td>(0.283)</td>
<td>(0.193)</td>
<td>(0.317)</td>
</tr>
<tr>
<td>Revision ( \times 1_{{t \geq t^*}} )</td>
<td>-1.111***</td>
<td>-1.245***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.379)</td>
<td>(0.341)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0875</td>
<td>0.271</td>
<td>-0.317***</td>
<td>-0.135*</td>
</tr>
<tr>
<td></td>
<td>(0.0696)</td>
<td>(0.185)</td>
<td>(0.0478)</td>
<td>(0.0690)</td>
</tr>
<tr>
<td>Constant ( \times 1_{{t \geq t^*}} )</td>
<td></td>
<td></td>
<td></td>
<td>-0.587***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.190)</td>
</tr>
<tr>
<td>Observations</td>
<td>197</td>
<td>58</td>
<td>139</td>
<td>197</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
The table shows the forecast error equation and the results of different regression models with structural breaks.

\[ \text{forecast error}_t = \alpha + (\beta_{\text{rev}} + \beta_{\text{rev},*} \times 1_{\{t \geq t^*\}}) \text{revision}_t + (\gamma + \gamma_* \times 1_{\{t \geq t^*\}}) \pi_{t-1,t-5} + \epsilon_{t}^{\text{rev}} \]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>CG Regression</td>
<td>Structural Break</td>
<td>Structural Break</td>
</tr>
<tr>
<td>Revision</td>
<td>1.220***</td>
<td>1.489***</td>
<td>1.476***</td>
</tr>
<tr>
<td></td>
<td>(0.248)</td>
<td>(0.316)</td>
<td>(0.296)</td>
</tr>
<tr>
<td>Revision \times 1{t \geq t^*}</td>
<td>-1.114***</td>
<td>-1.232***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.376)</td>
<td>(0.355)</td>
<td></td>
</tr>
<tr>
<td>\pi_{t-1,t-5}</td>
<td>0.00819</td>
<td>0.0103</td>
<td>-0.0482</td>
</tr>
<tr>
<td></td>
<td>(0.0340)</td>
<td>(0.0350)</td>
<td>(0.0352)</td>
</tr>
<tr>
<td>\pi_{t-1,t-5} \times 1{t \geq t^*}</td>
<td>-0.253***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0585)</td>
<td></td>
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</tr>
<tr>
<td>Observations</td>
<td>197</td>
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</tr>
</tbody>
</table>

Robust standard errors in parentheses

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Kohlhas & Walther (2021): correct for unbalancedness, number of forecasters

\[
\text{forecast error}_{jt} = \pi_{t+4} - F_{jt} \pi_{t+4}
\]

\[
\text{revision}_{t} = F_{t} \pi_{t+4} - F_{t-1} \pi_{t+4}
\]

Regress

\[
\text{forecast error}_{jt} = \beta_{\text{rev,ind}} \text{ revision}_t + u_{jt}
\]
Table: \( \text{forecast error}_{jt} = \alpha_{\text{ind}} + \beta_{\text{rev,ind}} \text{revision}_t + u_{jt} \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>revision</td>
<td>1.703***</td>
<td>1.131***</td>
<td>-0.0854</td>
<td>1.850***</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
<td>(0.200)</td>
<td>(0.138)</td>
<td>(0.188)</td>
</tr>
<tr>
<td>revision \times \mathbb{1}_{{t \geq t^*}}</td>
<td>-0.833***</td>
<td>-1.216***</td>
<td></td>
<td>-0.833***</td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td>(0.243)</td>
<td></td>
<td>(0.264)</td>
</tr>
<tr>
<td>Observations</td>
<td>6688</td>
<td>2294</td>
<td>4394</td>
<td>6688</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
Constant included

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
Table: forecast error_{jt} = \alpha_{\text{ind}} + \beta_{\text{rev,ind}} \text{ revision}_t + u_{jt}

<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
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<th>Column (3)</th>
<th>Column (4)</th>
<th>Column (5)</th>
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</thead>
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<tr>
<td>revision</td>
<td>1.703***</td>
<td>1.131***</td>
<td>-0.0854</td>
<td>1.850***</td>
<td>1.131***</td>
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<tr>
<td></td>
<td>(0.153)</td>
<td>(0.200)</td>
<td>(0.138)</td>
<td>(0.188)</td>
<td>(0.199)</td>
</tr>
<tr>
<td>revision \times 1{t \geq t^*}</td>
<td>-0.833***</td>
<td>-1.216***</td>
<td>-0.767***</td>
<td>-0.833***</td>
<td>-1.216***</td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td>(0.243)</td>
<td>(0.0571)</td>
<td>(0.264)</td>
<td>(0.243)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0392**</td>
<td>0.438***</td>
<td>-0.329***</td>
<td>-0.0719***</td>
<td>0.438***</td>
</tr>
<tr>
<td></td>
<td>(0.0183)</td>
<td>(0.0554)</td>
<td>(0.0138)</td>
<td>(0.0213)</td>
<td>(0.0554)</td>
</tr>
<tr>
<td>Constant \times 1{t \geq t^*}</td>
<td>-0.767***</td>
<td>-0.767***</td>
<td>-0.767***</td>
<td>-0.767***</td>
<td>-0.767***</td>
</tr>
<tr>
<td></td>
<td>(0.0571)</td>
<td>(0.0571)</td>
<td>(0.0571)</td>
<td>(0.0571)</td>
<td>(0.0571)</td>
</tr>
<tr>
<td>Observations</td>
<td>6688</td>
<td>2294</td>
<td>4394</td>
<td>6688</td>
<td>6688</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

* p < 0.10, ** p < 0.05, *** p < 0.01
Time-Varying Parameter Regression

\[ \pi_{t+4} - E_t \pi_{t+4} = \beta_t (E_t \pi_{t+4} - E_{t-1} \pi_{t+4}) + u_t \]
Disagreement

- Time series of “disagreement”
- Define *disagreement* at time $t$ as the cross-sectional standard deviation of forecasts at time $t$

$$\text{disagreement}_t = \sigma_i(F_{it}, \pi_{t+4})$$

- Disagreement fell around the mid-80s
Disagreement

• Concern: correlated with inflation level
• Show that for forecasters the inflation level is irrelevant
  * Underlying AR(p) inflation dynamics: individual

\[
\begin{align*}
F_{it} \pi_{t+3} & = \rho_1 F_{it} \pi_{t+2} + \rho_2 F_{it} \pi_{t+1} + \rho_3 F_{it} \pi_t + \gamma \pi_{t-1,t-5} + u_t \\
\end{align*}
\]

* Underlying AR(p) inflation dynamics: average

\[
\begin{align*}
\tilde{F}_{t} \pi_{t+3} & = \rho_1 \tilde{F}_{t} \pi_{t+2} + \rho_2 \tilde{F}_{t} \pi_{t+1} + \rho_3 \tilde{F}_{t} \pi_t + \gamma \pi_{t-1,t-5} + u_t \\
\end{align*}
\]

* Forecast error and revision

\[
\text{forecast error}_t = \beta \text{revision}_t + \gamma \pi_{t-1,t-5} + u_t
\]

* Forecast error autocorrelation

\[
\text{forecast error}_t = \beta \text{forecast error}_{t-1} + \gamma \pi_{t-1,t-5} + u_t
\]
<table>
<thead>
<tr>
<th></th>
<th>Individual forecasts</th>
<th>Average forecast</th>
<th>Error</th>
<th>Error</th>
</tr>
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<tr>
<td></td>
<td>AR(1)</td>
<td>AR(2)</td>
<td>AR(3)</td>
<td>AR(1)</td>
</tr>
<tr>
<td>$F_t \pi_{t+2}$</td>
<td>1.284***</td>
<td>1.435***</td>
<td>1.417***</td>
<td>1.356***</td>
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<tr>
<td></td>
<td>(0.0162)</td>
<td>(0.0476)</td>
<td>(0.0482)</td>
<td>(0.0190)</td>
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<tr>
<td>$F_t \pi_{t+1}$</td>
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<td>$F_t \pi_t$</td>
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<td>$error_{t-1}$</td>
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<td></td>
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<tr>
<td>$\pi_{t-1,t-5}$</td>
<td>0.00705</td>
<td>0.0119</td>
<td>0.0137*</td>
<td>-0.0299**</td>
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<td>(0.00909)</td>
<td>(0.00859)</td>
<td>(0.00819)</td>
<td>(0.0124)</td>
</tr>
</tbody>
</table>

Observations: 7,751 7,750 7,750 205 205 205 197 203

HAC robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Firm Problem
Short Derivation

- FOC wrt $P_{jt}$ and log-linearizing around the zero inflation steady-state

$$p_{jt}^* = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k \mathbb{E}_{jt} \left( mc_{j,t+k|t} + \mu \right), \quad \mu = \log \frac{\epsilon}{\epsilon - 1}$$

- Price equal to desired markup over (weighted) average of expected marginal costs

- In equilibrium
  * Individual marginal cost as a function of aggregate marginal cost:

$$mc_{j,t+k|t} = mc_{t+k} - \frac{\alpha \epsilon}{1 - \alpha} (p_{jt}^* - p_{t+k})$$

  * Aggregate demand = supply: $c_t = y_t$
  * Aggregate labor supply: $w_t - p_t = (\sigma + \phi)y_t$
  * Output in gap term, difference from natural rate: $\mu = -(\sigma + \phi) y^n_t, \tilde{y}_t = y_t - y^n_t$

- Recursive price-setting condition

$$p_{jt}^* = (1 - \beta \theta) \mathbb{E}_{jt} p_t + \frac{\kappa \theta}{1 - \theta} \mathbb{E}_{jt} \tilde{y}_t + \beta \theta \mathbb{E}_{jt} p_{j,t+1}^*$$
Marginal cost: cost of each unit of labor (wage) times labor needed to produce an additional unit of output

\[
mc_{j,t+k|t} = w_{t+k} - mpn_{j,t+k|t}
\]

\[
= w_{t+k} + \alpha n_{j,t+k|t} - \log(1 - \alpha)
\]

Define average marginal cost: \( mc_{t+k} = \int_{I_f} mc_{j,t+k} \, dj \)

\[
mc_{t+k} = w_{t+k} - mpn_{t+k}
\]

\[
= w_{t+k} + \alpha n_{t+k} - \log(1 - \alpha)
\]
We can write

\[
mc_{j,t+k|t} = mc_{t+k} + (w_{t+k} - w_{t+k}) + \alpha(n_{j,t+k|t} - n_{t+k})
\]

\[
= mc_{t+k} + \frac{\alpha}{1-\alpha}(v_{j,t+k|t} - v_{t+k})
\]

\[
= mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p^*_j - p_{t+k})
\]

* Inserting into price-setting condition,

\[
p^*_j = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^kE_{jt}[p_{t+k} + \frac{1-\alpha}{1-\alpha + \alpha\varepsilon}(mc_{t+k} - p_{t+k} + \mu)]
\]

\[
= (1-\beta\theta)E_{jt}p_t + (1-\beta\theta)\frac{1-\alpha}{1-\alpha + \alpha\varepsilon}E_{jt}(mc_t - p_t + \mu) + \beta\thetaE_{jt}p^*_{j,t+1}
\]
Firm Problem Solution

We can write

\[
mc_t - p_t = w_t + \alpha n_t - \log(1 - \alpha) \\
= \sigma c_t + (\varphi + \alpha)n_t - \log(1 - \alpha) \\
= \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)y_t - \log(1 - \alpha)
\]

\[\mu \equiv \text{markup under flexible prices}\]

\[
\mu = p_t - mc_t \\
= -w_t - \alpha n_t + \log(1 - \alpha) \\
= -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)y_t^n + \log(1 - \alpha)
\]

Defining \(\tilde{y}_t = y_t - y_t^n\)

\[
p^*_j = (1 - \beta \theta)\mathbb{E}_t p_t + (1 - \beta \theta)\frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon}\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\mathbb{E}_t \tilde{y}_t + \beta \theta \mathbb{E}_t p^*_{j,t+1}
\]
Solving Expectations
Obtaining Expectations

- Need to obtain expectations of output and price level
- Guess output and price dynamics

\[
\bar{y}_t = a_y p_{t-1} + b_y p_{t-2} + c_y v_t \\
p_t = a_p p_{t-1} + b_p p_{t-2} + c_p v_t \\
p^*_j = a_p p^*_{j,t-1} + b_p p^*_{j,t-2} + \frac{c_p}{1-\theta} x_{jt} - \frac{c_p \theta}{1-\theta} x_{j,t-1} \\
\]

- Using guesses, rewrite firm \( j \)'s policy function as beauty contest!

\[
p^*_j = \frac{\kappa \theta c_y}{1-\theta} \mathbb{E}_{jt} v_t + \frac{\kappa \theta b_y}{1-\theta} \mathbb{E}_{jt} p_{t-2} + \frac{\kappa \theta a_y}{1-\theta} \mathbb{E}_{jt} p_{t-1} + (1-\beta \theta) \mathbb{E}_{jt} p_t + \beta \theta \mathbb{E}_{jt} p^*_j, t+1 \\
\]

- Firm \( j \)'s action depends on her forecast of the fundamental, but also on my predictions of others’ actions
Obtaining Expectations

State-space representation

\[ Z_t = FZ_{t-1} + \bar{S}_{jt}, \quad x_{jt} = HZ_t + \bar{S}_{jt} \]

\[ Z_t = [v_t \quad p_t \quad p_{t-1} \quad p_{t-2}]', \quad S_{jt} = [\epsilon_t \quad u_{jt}]' \]

\[ F = \begin{bmatrix} \rho & 0 & 0 & 0 \\ \rho c_p & a_p & b_p & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \bar{S} = \begin{bmatrix} \sigma_{\epsilon} & 0 \\ \sigma_{\epsilon c_p} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 \end{bmatrix}', \quad \bar{S} = \begin{bmatrix} 0 \end{bmatrix}' \]
Obtaining Expectations

- Kalman filter

\[
E_{jt} Z_t = \tilde{E}_{j,t-1} Z_{t-1} + Kx_{jt}
\]  

(6)

\[
\sim = \begin{bmatrix}
\lambda & 0 & 0 & 0 \\
\rho - \lambda & \rho & a_p & b_p \\
1 - \lambda (a_p + \lambda b_p) & 1 & 0 & 0 \\
\lambda (\rho - \lambda) c_p & 0 & 1 & 0
\end{bmatrix}, \quad K = \begin{bmatrix}
1 - \frac{\lambda}{\rho} \\
\frac{\rho - \lambda c_p}{\rho (1 - \lambda a_p - \lambda^2 b_p)} \\
\frac{\rho (1 - \lambda a_p - \lambda^2 b_p)}{\lambda (\rho - \lambda) c_p} \\
\frac{\lambda (\rho - \lambda) c_p}{\rho (1 - \lambda a_p - \lambda^2 b_p)}
\end{bmatrix}
\]

and \( \lambda \) is the inside root of the following quadratic polynomial

\[
Q(z) = (z - \rho^{-1})(z - \rho) - \frac{\sigma_z^2}{\rho \sigma_u^2} z
\]
Using the lag operator, we can write (6) as

\[ \mathbb{E}_{jt} Z_t = (I - \tilde{L})^{-1} K x_{jt} \]

\[ = \tilde{(L)} x_{jt} \]

\[ \tilde{(L)} = \begin{bmatrix}
\frac{\rho - \lambda}{\rho(1-\lambda L)} \\
\frac{(\rho - \lambda)[1-\lambda(\rho a_p - (1-\rho \lambda) b_p)]c_p}{\rho(1-\lambda L)(1-a_p L - b_p L^2)(1-a_p \lambda - b_p \lambda^2)} \\
\frac{\rho(1-\lambda L)(1-a_p L - b_p L^2)(1-a_p \lambda - b_p \lambda^2)}{(\rho - \lambda)[\lambda^2 + (1-\rho \lambda + \lambda a_p)L + ((1-\rho \lambda)(1-\lambda a_p)-\lambda^2 b_p)L^2]c_p} \\
\frac{\rho(1-\lambda L)(1-a_p L - b_p L^2)(1-a_p \lambda - b_p \lambda^2)}{\rho(1-\lambda L)(1-a_p L - b_p L^2)(1-a_p \lambda - b_p \lambda^2)}
\end{bmatrix} \]
Obtaining Expectations

- Still need to find unknown \((a_p, b_p, c_p)\)!
- Recall firm \(j\) price-setting condition

\[
p_{jt}^* = \frac{\kappa \theta c_y}{1 - \theta} \mathbb{E}_{jt} v_t + \frac{\kappa \theta b_y}{1 - \theta} \mathbb{E}_{jt} p_{t-1} + \frac{\kappa \theta a_y}{1 - \theta} \mathbb{E}_{jt} p_t + (1 - \beta \theta) \mathbb{E}_{jt} p_t + \beta \theta \mathbb{E}_{jt} p_{j,t+1}^*
\]

- Have every necessary object since

\[
\mathbb{E}_{jt} p_{j,t+1}^* = a_p p_{jt}^* + b_p p_{j,t-1}^* + \frac{c_p}{1 - \theta} \rho \mathbb{E}_{jt} v_t - \frac{c_p \theta}{1 - \theta} \mathbb{E}_{jt} v_t
\]

- Plugging in our last result and the obtained expectations \(\mathbb{E}_{jt} Z_t\) we obtain a system of 3 equations that must hold \(\forall x_{jt}\)
- Obtain triplet \((a_p, b_p, c_p)\)!
- Given price dynamics, verify \(\tilde{\gamma}_t\) dynamics and solve for triplet \((a_y, b_y, c_y)\)
Model Dynamics
Proposition

Noisy information: inflation dynamics

\[ \pi_t = \delta \pi_{t-1} + \xi \pi_{t-2} + \psi \pi \Delta v_t \]

where \( \delta(\sigma_u, \Phi) \), \( \xi(\sigma_u, \Phi) \) and \( \chi(\sigma_u, \Phi) \) are scalars endogenous to information frictions \( \sigma_u \)

Corollary

In the frictionless limit (\( \sigma_u \rightarrow 0 \)), \( \delta \rightarrow 1 \), \( \xi \rightarrow 0 \) and \( \chi \rightarrow 1 \)
Proposition

Under noisy information price level dynamics are given by

\[ p_t = (\vartheta_1 + \vartheta_2)p_{t-1} - \vartheta_1 \vartheta_2 p_{t-2} - \psi_\pi \chi_\pi (\vartheta_1, \vartheta_2) v_t \]  

(7)

where \( \vartheta_1 \) and \( \vartheta_2 \) are the reciprocals of the two outside roots of the quartic polynomial

\[ \mathcal{P}(z) = -(\beta \theta - z)(1 - \theta z)(z - \rho)(1 - \rho z) \]

\[ - \tau z \left( (\beta \theta - z)(1 - \theta z) + z(1 - \theta)(1 - \beta \theta) \right) \]

\[ + z^2 \kappa \theta \frac{\vartheta_1 [\sigma(1 - \vartheta_2) + \phi_y](\vartheta_1 + \vartheta_2 - 1 - \phi_\pi) + (1 - \vartheta_2)(\phi_\pi - \vartheta_2)(\sigma + \phi_y)}{[\sigma(1 - \vartheta_1) + \phi_y][\sigma(1 - \vartheta_2) + \phi_y]} \]

\[ + z^3 \kappa \theta \frac{\vartheta_1 \vartheta_2 [\sigma(1 - \vartheta_1)(1 - \vartheta_2) - (\vartheta_1 + \vartheta_2 - 1 - \phi_\pi)\phi_y]}{[\sigma(1 - \vartheta_1) + \phi_y][\sigma(1 - \vartheta_2) + \phi_y]} \]

and \( \chi_\pi \) is a scalar endogenous to information frictions.
Proposition

Under noisy information output gap and price level dynamics are given by

\[
\tilde{y}_t = \frac{\vartheta_1 [\sigma(1 - \vartheta_2) + \phi_y] (\vartheta_1 + \vartheta_2 - 1 - \phi_{\pi}) + (1 - \vartheta_2) (\phi_{\pi} - \vartheta_2) (\sigma + \phi_y)}{[\sigma(1 - \vartheta_1) + \phi_y] [\sigma(1 - \vartheta_2) + \phi_y]} p_{t-1} \\
+ \frac{\vartheta_1 \vartheta_2 [\sigma(1 - \vartheta_1)(1 - \vartheta_2) - (\vartheta_1 + \vartheta_2 - 1 - \phi_{\pi}) \phi_y]}{[\sigma(1 - \vartheta_1) + \phi_y] [\sigma(1 - \vartheta_2) + \phi_y]} p_{t-2} - \psi_y \chi_y (\vartheta_1, \vartheta_2) v_t
\]

where \( \vartheta_1 \) and \( \vartheta_2 \) are the reciprocals of the two outside roots of the quartic polynomial \( P(z) \) and \( \chi_y \) is a scalar endogenous to information frictions.
Information Frictions

\[ \delta \in (1, \rho + \theta), \quad \delta'(\sigma_u) > 0, \]

\[ \xi \in (-\rho \theta, 0), \quad \xi'(\sigma_u) < 0 \]

\[ \chi \in (0, 1), \quad \xi'(\sigma_u) < 0 \]

Figure \( \delta \) and information frictions \( \sigma_u^2 \)

Figure \( \xi \) and information frictions \( \sigma_u^2 \)

Figure \( \chi \) and information frictions \( \sigma_u^2 \)
The role of $\theta$

- Information frictions affect $\vartheta_1$ and $\vartheta_2$ in opposing ways
- Want $\vartheta_2 \in (\theta, 1)$ not very sensitive
- Large value of $\theta$ limits this sensitivity
- Calvo price rigidity $\theta = 0.872$ implies a mean price duration of 7.8 quarters, upper range
- Macro-data: between 1-3.5 years [Gali (2015), Auclert, Rognlie & Straub (2020)]
The role of $\theta$

- Information frictions affect $\varrho_1$ and $\varrho_2$ in opposing ways
- Want $\varrho_2 \in (\theta, 1)$ not very sensitive
- Large value of $\theta$ limits this sensitivity
- Calvo price rigidity $\theta = 0.872$ implies a mean price duration of 7.8 quarters, upper range
- Macro-data: between 1-3.5 years [Gali (2015), Auclert, Rognlie & Straub (2020)]
- Depending on $\theta$: can explain 40%-100% of persistence fall
**Proposition**

The theoretical counterpart of the coefficient $\beta_{rev}$ is given by

$$\beta_{rev} = \frac{\lambda^3 \rho (1 - \theta_1 \lambda)(1 - \theta_2 \lambda)}{(1 - \lambda^4)(\rho - \lambda)} \left\{ \frac{\lambda (\lambda - \xi_1)(\lambda - \xi_2)(\lambda - \xi_3)(\lambda - \xi_4)}{(\lambda - \theta_1)(\lambda - \theta_2)} \right. - \left. (1 - \lambda^2) \left[ \frac{\theta_1 (\theta_1 - \xi_1)(\theta_1 - \xi_2)(\theta_1 - \xi_3)(\theta_1 - \xi_4)}{(1 - \lambda \theta_1)(\lambda - \theta_1)(\theta_1 - \theta_2)} + \frac{\theta_2 (\theta_2 - \xi_1)(\theta_2 - \xi_2)(\theta_2 - \xi_3)(\theta_2 - \xi_4)}{(1 - \lambda \theta_2)(\lambda - \theta_2)(\theta_1 - \theta_2)} \right] \right\}$$

where

- $\delta = \theta_1 + \theta_2$ and $\xi = -\theta_1 \theta_2$
- $\lambda$ is the inside root of the quadratic polynomial $Q_1(z) = (1 - \rho z)(z - \rho) + \frac{\sigma^2 \xi}{\sigma^2_u} z$
- $\{ \xi_1, \xi_2, \xi_3, \xi_4 \}$ are the reciprocals of the roots of the quartic polynomial

$$Q_2(z) = \phi_0 + \phi_1 z + \phi_2 z^2 + \phi_3 z^3 + \phi_4 z^4$$

where $\phi_0 = c_p$, $\phi_1 = \left(\frac{1}{\lambda} - \frac{1}{\rho}\right) c_p$, $\phi_2 = \frac{(\rho - \lambda)c_p}{\lambda^2 \rho}$, $\phi_3 = \frac{(\rho - \lambda)c_p}{\lambda^3 - \theta_1 - \theta_2 + \lambda \theta_1 \theta_2}$, and $\phi_4 = -\lambda^3 + \lambda^4 \theta_2 + \lambda^4 \theta_1 - \theta_1 \theta_2 [\lambda - (1 - \lambda^4)\rho]$.
Proposition

The theoretical counterpart of the coefficient $\beta_{\text{rev}}$ is given by

$$
\beta_{\text{rev}} = \frac{\lambda}{\rho - \lambda} \left[ (1 + \lambda)(\delta + \lambda \xi) - 1 - \frac{(\rho - \lambda)}{1 - \lambda(\delta + \lambda \xi)} \right] \left[ \lambda \xi + \frac{\delta + \lambda \xi - 1}{1 - \lambda} \right]
$$

where $\lambda$ is the inside root of the following quadratic polynomial

$$Q(z) = (1 - \rho z)(z - \rho) + \frac{\sigma^2_\varepsilon}{\sigma^2_u} z$$
Wedge Phillips Curve

- Noisy information pre-1985

Proposition

Suppose we want to reproduce the noisy information dynamics in a FIRE setting. Guess that inflation dynamics follow

$$\pi_t = \omega_1 \pi_{t-1} + \omega_2 K\tilde{y}_t + \omega_3 \beta E_t \pi_{t+1}$$

The above wedge Phillips curve produces identical dynamics for certain values of $$(\omega_1, \omega_2, \omega_3) \in [0, 1]^3$$

- $\omega_1 \in (0, 1)$: anchoring
- $\omega_3 \in (0, 1)$: myopia
Proposition

Suppose we want to reproduce the noisy information dynamics in a FIRE setting. Guess that inflation dynamics follow

\[ \pi_t = \omega_1 \pi_{t-1} + \omega_2 \kappa \tilde{y}_t + \omega_3 \beta E_t \pi_{t+1} \]

The above wedge Phillips curve produces identical dynamics for certain values of \((\omega_1, \omega_2, \omega_3) \in [0, 1]^3\)

- \(\omega_1 \in (0, 1)\): anchoring
- \(\omega_3 \in (0, 1)\): myopia
- Post 1985: \(\omega_1 = 0, \omega_2 = \omega_3 = 1\)

\[ \pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1} \]
### Table Simulated Wedge Phillips Curve

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HAC Robust standard errors in parentheses

Instruments: four lags of inflation and output gap

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
## Table Regression table

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Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$