Heterogeneity in Imperfect Inflation Expectations: Theory and Evidence from a Novel Survey

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Disclaimer

This paper uses data from the Bundesbank-Online-Panel-Households. The results published and the related observations and analysis may not correspond to results or analysis of the data producers.
Decomposing Uncertainty in Inflation Expectations

Suppose that each household $i$ perceives inflation to be AR(1):

$$\pi_{t+1} = \tilde{\rho}_i \pi_t + \varepsilon_{t+1}$$

Uncertainty in inflation expectations can then be decomposed into:

$$\tilde{\text{Var}}_{i,t}(\pi_{t+1}) = \tilde{\rho}_i^2 \cdot \tilde{\text{Var}}_{i,t}(\pi_t) + \tilde{\sigma}_{\varepsilon,i}^2$$

- Uncertainty about future inflation
- Perceived persistence
- Uncertainty about current inflation
- Uncertainty from shocks

**Our contribution:** We add novel questions to Bundesbank survey to elicit $\tilde{\rho}_i$ and $\tilde{\text{Var}}_{i,t}(\pi_t)$ at household level.
Survey Questions: Uncertainty in Inflation Perceptions

Respondents have already been asked for a point estimate of the current inflation rate. We then ask:

Now we would like to know how certain you are about your information on the inflation rate or deflation rate over the past 12 months ([Value of point estimate])%.

In your opinion, how likely is it that the inflation rate has been between [Low inflation level]% and [High inflation level]% over the past twelve months?

___ percent

______________________________
Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Results: Perception Uncertainty

Uncertainty about current inflation is typically much lower than uncertainty about future inflation.

Figure: CDF of $\sqrt{\tilde{V}_{ar_i,t}(\pi_t)}$ and $\sqrt{\tilde{V}_{ar_i,t}(\pi_{t+1})}$

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Respondents split into three groups. Each group shown a different hypothetical scenario, similar to Andre et al. (2022).

**Group 1:** Imagine the following hypothetical situation: Due to an unexpected economic event, the inflation rate increased by one percentage point in the past year.

Group 2 shown a supply shock scenario (oil supply shock), group 3 a demand shock scenario (government spending shock).
Group 2: Imagine the following hypothetical situation: Due to unexpected problems with local production technology in the Middle East, the price of crude oil rose in the past year, causing the inflation rate to rise by one percentage point.

Group 3: Imagine the following hypothetical situation: Due to increased defense spending, government spending rose unexpectedly more than usual in the past year, causing the inflation rate to rise by one percentage point. The change is temporary and occurs even though the government’s assessment of national security or economic conditions has not changed. In addition, taxes do not change in response to the spending program.

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
All respondents are then asked:

*In this situation, would you adjust your inflation expectations for the next 12 months as stated in the first part of the questionnaire? If so, to what extent?*

1) Yes, from [Value of point estimate]% to ___%
2) No

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Results: Perceived Persistence

Average perceived persistence is roughly in line with data ($\approx 0.2$), but responses are very heterogeneous.

Figure: CDF of perceived persistence, by hypothetical scenario

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Theory: Law of Motion for Inflation Expectations

Assume households receive noisy signals about current inflation:

\[ s_{i,t} = \pi_t + q_{i,t} \]

The law of motion for inflation expectations at individual level is:

\[ \tilde{E}_{i,t} \pi_{t+1} = (1 - \chi_i) \tilde{\rho}_i \tilde{E}_{i,t-1} \pi_t + \chi_i \tilde{\rho}_i (\pi_t + q_{i,t}) \]

The Kalman gain \( \chi_i \) is given by:

\[ \chi_i = 1 - \frac{V_i^p}{V_i^f} \]

\( V_i^p \) is steady-state \( \tilde{\text{Var}}_{i,t} (\pi_t) \), and \( V_i^f \) is steady-state \( \tilde{\text{Var}}_{i,t} (\pi_{t+1}) \).
Results: Implied Kalman Gain

Implied Kalman gain is high on average ($\approx 0.8$), but responses are again very heterogeneous.

Figure: CDF of $\chi_i$

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Suppose that true process for inflation is AR(1):

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t$$

Suppose inflation and inflation expectations start in steady state, and there is a one percentage point shock to inflation in $t = 0$.

We map IRFs of aggregate inflation expectations and consumption in three cases:

1. Full information rational expectations
2. Representative agent model based on average $\chi_i$ and $\tilde{\rho}_i$
3. Heterogeneity, using full joint distribution of $\chi_i$ and $\tilde{\rho}_i$
Theory: Partial-Equilibrium Consumption Response

Standard infinite horizon consumption-savings problem:

\[ \hat{c}_{i,t} = \sum_{h \geq 0} \beta^h \left( (1 - \beta) \hat{E}_{i,t} \hat{y}_{i,t+h} - \beta \gamma^{-1} \hat{E}_{i,t} \hat{i}_{t+h} + \beta \gamma^{-1} \hat{E}_{i,t} \pi_{t+h+1} \right) \]

Assume no change in nominal interest rate or income to isolate effect of change in inflation expectations. Assume \( \hat{c}_{i,t} = 0 \) for hand-to-mouth consumers.

In this model, consumers are very forward looking, so the consumption response is highly convex in \( \hat{\rho}_i \).

- **Implication:** Heterogeneity in \( \hat{\rho}_i \) amplifies partial equilibrium consumption response to shock.
Implied IRFs: Aggregate Inflation Expectations

Expectations IRFs look similar across three cases, but...

Figure: IRF of $\hat{E}_t \pi_{t+1}$

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Implied IRFs: Aggregate Consumption

...consumption response is an order of magnitude larger under heterogeneity, and much more persistent.

Figure: IRF of $\hat{c}_t$

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Summary

- We use novel survey data to identify (i) uncertainty in inflation perceptions, and (ii) perceived persistence of inflation.
- Together with existing survey data and some modelling assumptions, we can then identify laws of motion for inflation expectations at individual level.
- Based on averages alone, model-implied response of expectations and consumption to inflation is small and transitory.
- Accounting for the heterogeneity in the data, consumption response is an order of magnitude larger and far more persistent.
Appendix
Results: Q1 Raw Responses

A large fraction of consumers are 100% confident current inflation lies within the specified interval.

Figure: CDF of responses to Q1

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Results: Uncertainty from Shocks

Most uncertainty in inflation expectations stems from uncertainty about future shocks.

Figure: CDF of $\tilde{\sigma}_{\varepsilon,i}$

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Results: Correlations

**Table**: Pair-wise correlations of subjective law of motion elements.

<table>
<thead>
<tr>
<th></th>
<th>$SD_i(\pi_{t+1})$</th>
<th>$SD_i(\pi_t)$</th>
<th>$\tilde{\rho}_i$</th>
<th>$SD_i(\varepsilon_{t+1})$</th>
<th>$\chi_i$</th>
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</thead>
<tbody>
<tr>
<td>$SD_i(\pi_{t+1})$</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SD_i(\pi_t)$</td>
<td>0.473***</td>
<td>1.000</td>
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<tr>
<td>$\tilde{\rho}_i$</td>
<td>0.028</td>
<td>-0.046**</td>
<td>1.000</td>
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<td></td>
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<tr>
<td>$SD_i(\varepsilon_{t+1})$</td>
<td>0.988***</td>
<td>0.440***</td>
<td>-0.030</td>
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<tr>
<td>$\chi_i$</td>
<td>0.305***</td>
<td>-0.402***</td>
<td>0.073***</td>
<td>0.327***</td>
<td>1.000</td>
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</table>

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Table: Regressions of components of subjective laws of motion on household characteristics.

<table>
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<tr>
<th></th>
<th>(1) $\log(SD_i(\pi_{t+1}))$</th>
<th>(2) $\log(SD_i(\pi_t))$</th>
<th>(3) $\log(SD_i(\varepsilon_{t+1}))$</th>
<th>(4) $\log(\chi_i)$</th>
<th>(5) $\tilde{\rho}_i$</th>
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<td>Hand-to-mouth</td>
<td>0.0200</td>
<td>0.1374**</td>
<td>0.1477***</td>
<td>0.0951**</td>
<td>0.4072**</td>
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<td></td>
<td>(0.0360)</td>
<td>(0.0541)</td>
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<td>Liquid wealth</td>
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<td>-0.0002**</td>
<td>0.0002**</td>
<td>0.0002***</td>
<td>-0.0013**</td>
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<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
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<tr>
<td></td>
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<td>(0.0000)</td>
<td>(0.0000)</td>
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<td>Other wealth</td>
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<td>-0.0000</td>
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<td>(0.0007)</td>
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<td>0.0001</td>
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<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0001)</td>
<td>(0.0006)</td>
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<td>log(income)</td>
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<td>$-0.1247^{***}$</td>
<td>$-0.1736^{***}$</td>
<td>$-0.0019$</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
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<td>4382</td>
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<td>2292</td>
<td>2024</td>
<td>3194</td>
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</table>

Source: Bundesbank-Online-Panel-Households, November 2021 wave.
Theory: Convexity of Consumption Responses

Formally, the aggregate consumption response in $t = 0$ is:

$$\hat{c}_0 = \beta \gamma^{-1} \left( E \left[ \frac{\tilde{\rho}_i}{1 - \beta \tilde{\rho}_i} \right] E[\chi_i] + \text{Cov} \left( \frac{\tilde{\rho}_i}{1 - \beta \tilde{\rho}_i}, \chi_i \right) \right)$$

Note that $\frac{\tilde{\rho}_i}{1 - \beta \tilde{\rho}_i}$ is convex in $\tilde{\rho}_i$. By Jensen’s inequality, heterogeneity in $\tilde{\rho}_i$ (for a given $E[\tilde{\rho}_i]$) leads to a larger aggregate consumption response.
Results: Consumption IRFs by Shock

Amplification is greatest for supply shocks, because of their higher perceived persistence.

Figure: IRF of $\hat{c}_t$ by hypothetical scenario

Source: Bundesbank-Online-Panel-Households, November 2021 wave.