Abstract

We present a model in which each of a number of bankers chooses to set up a traditional or a shadow bank. Shadow banks escape the costly regulation traditional banks must comply with, but do not have access to deposit insurance, which traditional banks can tap in a crisis. We show that in equilibrium traditional and shadow banks coexist. In a crisis, shadow banks repay their creditors by selling assets to traditional banks at fire-sale prices. The larger the size of traditional relative to shadow banks, the higher these asset prices, and thus the higher the incentive to set up a shadow bank in the first place. In a crisis, traditional banks purchase shadow banks' assets by issuing new deposits backed by deposit insurance. The larger the size of shadow relative to traditional banks, the lower these asset prices and thus the higher the incentive to set up a traditional bank in the first place. The model implies that an increase in deposit insurance decreases the relative size of the traditional banking system. In equilibrium, the shadow banking sector is shown to be too large compared to the social optimum. Our model is consistent with several facts from the 2007 financial crisis: some assets and (deposit-like) liabilities migrated from shadow banks to traditional banks, and shadow bank assets were sold at fire sale prices.

Keywords: Traditional banking, Shadow banking, Financial crisis, Deposit insurance

JEL Codes: E32, E44, E61, G01, G21, G23, G38.
1 Introduction

"Shadow banks" are market-based institutions ranging from money market funds to asset-backed securities issuers, performing bank-like activities outside of the traditional regulated banking system. Those institutions borrow not only from ultimate creditors (households), but also from each other, supplying credit through more or less complex intermediation chains. Shadow banks now account for about a quarter of total financial intermediation worldwide.\(^1\) The regulatory arbitrage view invokes regulatory costs to explain why financial intermediation has migrated away from traditional banks and into shadow banks, failing to explain why traditional and shadow banks coexist in the market place. The collapse of shadow banking in 2007 threatened traditional banks’ stability, raising important questions about the interconnection between both types of banks.

We present a model in which each of a number of bankers chooses to set up a traditional or a shadow bank. Shadow banks escape the costly regulation traditional banks must comply with, but do not have access to deposit insurance, which traditional banks can tap in a crisis. We show that in equilibrium traditional and shadow banks coexist. In a crisis, shadow banks repay their creditors by selling assets to traditional banks at fire-sale prices. The larger the size of traditional relative to shadow banks, the higher these asset prices, and thus the higher the incentive to set up a shadow bank in the first place. In a crisis, traditional banks purchase shadow banks’ assets by issuing new deposits backed by deposit insurance. The larger the size of shadow relative to traditional banks, the lower these asset prices and thus the higher the incentive to set up a traditional bank in the first place. The model implies that an increase in deposit insurance decreases the relative size of the traditional banking system. In equilibrium, the shadow banking sector is shown to be too large compared to the social optimum. Our model is consistent with several facts from the 2007 financial crisis: some assets and (deposit-like) liabilities migrated from shadow banks to traditional banks, and shadow bank assets were sold at fire sale prices.

We consider a model with three dates 0, 1 and 2. At date 0, a number of bankers can set up a traditional bank or a shadow bank. Both types of banks issue short-term debt to households to invest in risky assets that payoff at date 2. At date 1, news about risk asset returns arrives and there are two possible states of nature. Either a crisis occurs, during which expected asset returns at date 2 are low and uncertain, with a non-zero probability that they will be zero. Or no crisis occurs and asset returns will be high and certain at date 2.

Traditional and shadow banks differ as follows. On the one hand, shadow banks escape the costly regulation traditional banks must comply with. Indeed, most observers agree that the growth of shadow banking has been largely driven by regulatory arbitrage. On the other hand, only traditional banks can access deposit insurance, which is particularly valuable in a crisis. Indeed, the recent crisis provided a striking illustration of the need for deposit insurance in a crisis. We thus assume that shadow banks do not have access to deposit insurance, which traditional banks can tap in a crisis.

When bankers initially choose to set up a traditional or a shadow bank, they trade off the costs and

\(^1\)This estimate is in terms of credit intermediation (see IMF, 2014). For empirical descriptions of shadow banking, see Pozsar et al. (2013) for the United States, ESRB (2016) for the European Union, IMF (2014) and FSB (2015) for global estimates. Globally, shadow banks’ assets were worth $80 trillion in 2014, up from $26 trillion more than a decade earlier (FSB (2015)).
benefits associated with each type of bank. If they set up a shadow bank, bankers escape the costly regulation traditional banks must comply with. If they set up a traditional bank, bankers must comply with costly regulation.

Absent a secondary market for assets in a crisis, the trade-off is the following. On the one hand, shadow banks cannot rely on traditional banks to purchase their assets in a crisis, therefore they are unable to issue short-term debt before a crisis. On the other hand, traditional banks use deposit insurance to roll-over their short-term debt in a crisis. However, access to deposit insurance comes at the cost of costly regulation. Expected profits are relatively low in the traditional banking sector. In this world, traditional and shadow banks are then substitutes, and bankers trade-off expected profits from unlevered shadow banking versus traditional banking without the benefits of purchasing fire-sold assets in a crisis. One type of bank has higher expected profits and the two bank types do not coexist.

Now, if there is a secondary market for assets in a crisis, the trade-off become the following. In a crisis, expected asset returns are low with a positive probability that they will be zero. On the one hand, shadow banks cannot use risky assets to back short-term debt in a crisis because this debt has to be repaid no matter whether asset returns are positive or not and shadow banks do not have access to deposit insurance. In a crisis, shadow banks sell their assets to repay their creditors. As a result, before a crisis, shadow banks can finance their asset purchases partly with short-term debt while escaping costly regulation. The amount of short-term debt that shadow banks can issue before a crisis depends on asset prices in a crisis. On the other hand, traditional banks can issue short-term debt in a crisis to purchase assets from shadow banks, which they back by deposit insurance. Traditional banks’ ability to issue short-term debt is limited, therefore shadow banks’ assets trade at a discount. Despite the fact that they must comply with costly regulation, traditional banks benefit from the advantage that they earn a profit from asset purchases at fire-sale prices from shadow banks in a crisis.

In equilibrium, bankers must be indifferent between setting up a traditional or a shadow bank. This pins down asset prices and thus the relative size of the traditional and shadow banking sectors. The larger the size of traditional banks relative to shadow banks, the higher asset prices in a crisis, and the higher the amount of short-term debt that shadow banks can issue before a crisis. Therefore the more traditional banks relative to shadow banks, the higher shadow banks’ expected profits and the higher bankers’ incentive to set up a shadow bank in the first place. Conversely, the larger the size of shadow banks relative to traditional banks, the lower asset prices in a crisis and the higher the fire-sale discount at which traditional banks purchase shadow banks’ assets in a crisis. Therefore the more shadow banks relative to traditional banks, the higher traditional banks’ expected profits and the higher bankers’ incentive to set up a traditional bank in the first place. In that sense, traditional and shadow banks form an ecosystem.

Bankers are banks’ equity holders in our model. Since debt holders are different from equity holders, a natural question that arises is the following: why don’t equity holders take advantage of debt holders by shifting risk to them? In our paper, since there is no further asset return uncertainty at date 1 when there is no crisis, no bank wants to shift risk in that state. At date 1 when there is a crisis, shadow banks

\[\text{We think of the guarantee fund as a government entity. Therefore this limit captures some form of limited fiscal capacity that prevents the guarantee fund from insuring too large an amount of traditional banks' debt.}\]
cannot shift risk because they cannot issue debt to purchase risky assets. Traditional banks neither have an incentive to shift risk because of actuarially fair deposit insurance: if they take more risk the insurance premium rises accordingly, leaving them indifferent between doing so or not.

Our model is consistent with several facts from the 2007 financial crisis, which we document in section 2. First, during the financial crisis almost $600 billion of deposits and borrowings went into the largest traditional banks in 2008q3. This fact is consistent with our model, whereby traditional banks tap deposit insurance in a crisis to issue short-term debt. This happened in less than a month, concomitantly to a wide run on the shadow banking system. In the model, shadow banks repay their creditors while being unable to issue more short-term debt, consistent with the stylized fact that shadow banks’ short-term debt collapsed during the crisis.

Second, deposit flows were mirrored by asset flows, with approximately $800 billion assets out of shadow banks and $550 billion into traditional banks from 2007q4 to 2009q1. We further show that traditional banks purchased assets from shadow banks by issuing insured deposits. This is again consistent with our model, in which absent deposit insurance for traditional banks, neither type of bank can back short-term debt in a crisis because asset returns then are low and uncertain, with a positive probability that they might be zero. We document that it is indeed by issuing insured deposits, as opposed to brokered deposits, that traditional banks purchased assets during the crisis.

Third, some assets were sold at fire sale prices; notably mortgage-backed government-agency securities. Asset fire sales are key in our model in that in equilibrium, for bankers to be indifferent between setting up a traditional or a shadow bank, asset purchases by traditional banks in a crisis must compensate them for regulatory costs compliance. Without asset fire sales from shadow to traditional banks, bankers would obtain higher expected profits by operating unregulated shadow banks and the two bank types could not coexist.

Next, we analyze the implications of changes in the level of deposit insurance for traditional banks. Although arbitrage of regulatory costs has been an important feature of the banking industry since the first Basel accords (see e.g. Hanson et al., 2011), little is known about the effects of changes in deposit insurance on the relative size of traditional versus shadow banks. We find two competing effects. An expansion in traditional banks’ ability to issue deposits in a crisis is that traditional banks’ relative advantage is increased: Traditional banks have access to a higher level of deposit insurance, which increases their leverage at all times, hence increasing their profits. This makes traditional banking relatively more profitable than shadow banking, thereby increasing bankers’ incentives to set up a traditional bank.3

The reverse reasoning holds true for a decrease in traditional banks’ ability to issue deposits in a crisis. The reason why we consider small changes in traditional banks’ support in a crisis is that large changes in traditional banks’ support in a crisis wipe out either type of bank from the market, which is an effect already emphasized in existing models of shadow banking as regulatory arbitrage (see e.g. Plantin (2015), Ordonez (2013), Harris et al. (2015)).

3The second effect follows from the fact that shadow banks benefit from traditional banks’ access to the guarantee fund through the (secondary) asset market in a crisis. Expanding traditional banks’ guarantee in a crisis increases the support that they provide to shadow banks, which in turn increases the price at which assets are sold on the secondary market. This increases shadow banks’ profits and bankers’ incentives to set up a shadow bank.
We show that the latter effect dominates the former. Here is the intuition behind this result. First, remark that shadow banks finance their assets using a mix of short-term debt and equity. Traditional banks extract profits from fire-sold assets because the amount of assets they purchase from shadow banks is higher than the amount of short-term debt issued by shadow banks. When deposit insurance expands, the quantity of short-term debt issued by shadow banks adjusts and more assets are purchased for a constant level of shadow bank equity. Thus leverage increases in the shadow banking sector. When deposit insurance expands, the transfer from traditional to shadow banks due to asset purchases in a crisis involves assets that are financed with a shrinking fraction of shadow bank equity. The cost of the asset transfer for traditional banks evolves accordingly but the payoff from seized assets increases relatively less because a smaller fraction of shadow bank equity was used to finance these assets in the first place. Expanding the level of deposit insurance induces a dilution of traditional banks’ profits from purchasing fire-sold assets. Overall, expanding support to traditional banks in a crisis increases asset prices to such an extent that more bankers choose to enter the shadow banking sector ex-ante.

Last, we consider the normative implications of our analysis, comparing the social planner’s allocation to bankers’ decentralized allocation between traditional and shadow banking technologies. We find that asset fire sales are at the root of a pecuniary externality whereby bankers operate too many shadow banks in equilibrium. Bankers fail to internalize that operating a shadow bank reduces the support from traditional banks in a crisis, hence reducing other shadow banks’ ability to lever before a crisis.

The paper proceeds as follows. In Section 2 we discuss the existing literature and document the following facts from the crisis, which our model can replicate: (i) liabilities transfer from shadow to traditional banks, (ii) assets transfer from shadow to traditional banks, and (iii) fire sales of assets. Section 3 presents the model, and we analyze the possible coexistence between shadow and traditional banks in Section 4. In Section 5 we discuss the substitution and complementarity effects, and the implications of our model. Section 6 concludes.

2 Stylized facts and literature review

In this section, we discuss the literature in light of three stylized facts. We use data from the Financial Accounts of the United States (henceforth FAUS), the Federal Reserve H8 Releases and the quarterly Call Reports.

For instance, a money market fund (MMF) investing in safe assets while issuing risk-free claims to households is a shadow bank. More sophisticated chains often include institutions such as “mortgage pools” or “ABS issuers” which use risky assets to back their liabilities. In that case, some institution along the intermediation chain has to contribute financing in the form of equity. Still, at each end of the chain, shadow banks perform traditional banking activities: they invest in risky assets by leveraging their equity with risk-free claims issued to households.4

In the FAUS data, we define traditional banks as the private depository institutions (L.110). Those institutions are composed of U.S.-chartered depository institutions (L.111), foreign banking offices (L.112),

4For similar approaches to shadow banking, see Hanson et al. (2015), Shin (2010) or Annex 2.2 in IMF (2014).
banks in U.S.-affiliated areas (L.113) and credit unions (L.114). Although our stylized facts do not rely on a precise definition of shadow banking using the FAUS, quantitative results depend on which institutions we identify as part of the shadow banking sector. As explained in the introduction, we define shadow banks as chains of market-based transactions among legal institutions which, taken together, perform maturity transformation activities comparable to that of traditional banks. We define shadow banks in the data as the sum of money market mutual funds (L.121), mutual funds (L.122), issuers of asset-backed securities (L.127) and security brokers and dealers (L.130). Finally, we use Krishnamurthy and Vissing-Jorgensen (2015)'s definition of short-term debt from the FAUS, 60% of which is composed of small time and savings deposits in the 2007-09 period. We provide details on data construction in Appendix A.1.

Existing theories of traditional and shadow banking emphasize the substitutability between the two. Given that shadow banks are not subject to the regulations that pertain to traditional banks, these regulations might spur financial intermediation into shadow banking to exploit regulatory arbitrage. This view emphasizes the regulatory costs of traditional banks, failing to explain why traditional and shadow banks coexist and omitting the fact that the two bank types behaved differently in the crisis.

In this paper, we argue that traditional banks still compete with but also complement shadow banks, thanks to their ability to issue deposits in a crisis using deposit insurance. Diamond and Dybvig (1983) is the seminal paper providing a rationale for traditional banks’ deposit insurance that is based on the elimination of depositors’ incentives to run their bank. Merton (1995) and Rajan (1998a,b) are early discussions questioning the future of traditional banks in light of increased competition from other types of banks in the modern institutional environment.

To the best of our knowledge, this paper is the first to provide a theory of the coexistence between traditional and shadow banks based on their interactions in a crisis via gains from asset trade. An exception is LeRoy and Singhania (2017), who provide a theory of the interaction between traditional and shadow banks driven by traditional banks gaming deposit insurance outside of a crisis, complementing our paper well.

2.1 Fact 1: Liabilities flow from shadow to traditional banks

In the early phase of the 2007 financial crisis, investors stopped rolling over shadow banks’ short-term funding. Gorton and Metrick (2011) and Copeland et al. (2014) document investors’ run on their major yet unstable source of funding: the sale and repurchase market (the "repo" market). Another important fact we emphasize is the deposit inflow on traditional banks’ balance-sheets. Table 1 in Appendix A.2 shows the evolution of short-term debt for traditional and shadow banks from 2006q4 to 2011q1. It is apparent from Table 1 that there was a concomitant run on shadow banks and an inflow of short-term debt into traditional banks starting 2008q3. This inflow of deposits in turbulent times is the risk management motive emphasized in Kashyap et al. (2002) to explain why traditional banks combine demand deposits with loan commitments or lines of credit: In a crisis, borrowers draw down on their

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5Earlier descriptive studies adopt similar approaches to shadow banks, see e.g. Pozsar et al. (2013), or Adrian and Shin (2010).
6Other early discussions of the evolution of the financial landscape are Boyd and Gertler (1994) and James and Houston (1996).
credit lines while investors seek a safe haven for their wealth, turning to traditional banks because these latter provide insurance due to the government guarantee on their deposits.

Gatev and Strahan (2006) emphasize that it is traditional banks’ access to federal deposit insurance that causes economy’s savings to move into traditional bank deposits during times of aggregate stress, providing banks with the unique ability to hedge against systematic liquidity shocks. Nevertheless, as can be seen on Figure 11 in Appendix A.2 and documented in Acharya and Mora (2015), it was not until the U.S. government’s intervention just before the Lehman failure on September 15, 2008 that deposit flew into traditional banks. Core deposits eventually increased by close to $800 billion by early 2009. He et al. (2010) find similar results. Weekly times series in Figure 11 show a sudden $600 billion deposits and borrowings inflow into the largest US traditional banks in just a few days, from September 10th to October 1st, 2008.

The flow of deposits into traditional banks illustrates the fact that not all entities of the U.S. financial sector deleveraged in the crisis. Ang et al. (2011) show that hedge fund leverage decreases prior to and during the financial crisis from mid-2007 onwards, He et al. (2010) show that leverage of banks and investment banks continues to increase. This helps to put deleveraging into perspective. At the worst periods of the financial crisis in late 2008, hedge fund leverage is at its lowest while the leverage of banks is at its highest. Although traditional banks issued new equity during the crisis, in Appendix A.2.3 we show the evolution of traditional banks’ market and book equity over the crisis. We find that traditional banks did not issue enough new equity during the crisis to compensate for their market losses. As a result, the deposit flow into traditional banks increased their market leverage in the crisis.

2.2 Fact 2: Asset flow from shadow to traditional banks

Figure 13 in Appendix A.2.4 illustrates that assets flew out of shadow banks and into traditional banks during the financial crisis. Although our data does not allow us to identify whether these changes were due to changes in the value of assets or changes in ownership, empirical work by He et al. (2010) and Bigio et al. (2016) provide estimates of the amount of assets that were transferred from shadow to traditional banks during the crisis. From 2007q4 to 2009q1, He et al. (2010) find that shadow banks decreased their holdings of securitized assets by approximately $800 billion while traditional banks increased theirs by approximately $550 billion. Looking at the wider period from 2007q1 to 2013q1 and considering total asset holdings, Bigio et al. (2016) document a net asset outflow of $1702 billion out of shadow banks and an asset inflow of $1595 billion into traditional banks.

The main argument that explains why the shadow banking system developed is that traditional banks create off balance-sheets entities, because holding loans on balance sheets is not profitable for them see e.g. Gorton and Metrick (2011), Acharya et al. (2013). For instance special conduits are comparable to regular banks in many ways, and they often are managed by traditional banks. The flip side

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7This explains why there is no evidence that funds flowed into the banking system when spreads widened during the 1920s, prior to the expansion of the federal safety net with the creation of federal deposit insurance.
8See Baron (2016) for evidence of banks’ countercyclical equity issuance.
9Another important aspect of this asset transfer is the sizable purchase of assets from the Federal Reserve, which balance sheets increased by approximately $1954 billion, as calculated in Bigio et al. (2016).
10Pozsar et al. (2013) dubs shadow banks managed by traditional banks “internal shadow banking.”
of this off-balance sheet leverage in the shadow banking sector is that liquidity guarantees are provided by traditional banks. Acharya et al. (2013) show that investors in conduits covered by guarantees were repaid in full. This implies a monetary transfer from the traditional to the shadow banking system in the crisis, and the mirror asset transfer from shadow to traditional banks evidenced in Figure 13.

A group of theories relates to traditional banks’ regulation and their coexistence with shadow banks. Hanson et al. (2015) are interested in the implications of traditional versus shadow banking businesses in terms of the assets that are held by financial intermediaries. Plantin (2015) studies optimal bank capital regulation in the presence of shadow banking, and finds that the optimal regulation needs not be in line with current regulatory reforms. Ordonez (2013) proposes a model in which reputational concerns are an effective disciplining device in the shadow banking sector. When reputation concerns are weak, banks can only operate using traditional banking. Harris et al. (2015) develop a model where capital requirements reduce banks’ risk-taking incentives while lowering their funding capacity, and discuss the cyclicality of optimal bank capital regulation in light of the amount of capital in the economy. Gor- nicka (2016) and Luck and Schempp (2014) present models where a crisis in the shadow banking sector transmits to the traditional banking sector through guarantees to shadow banks.

One main testable prediction of our theory that we take to the data is that traditional banks are able to purchase assets from shadow banks in a crisis, insofar as they benefit from a guarantee on their deposits. This guarantee indeed enables them to attract deposits precisely when shadow banks have to repay their creditors. We use the Call Reports and He et al. (2010)’s estimates to test this prediction in the cross-section of traditional banks during the crisis. Details about the sample construction and results are in Appendix A.3. We find support for the central mechanisms of our theory: Traditional banks purchased mortgage-backed securities in the crisis, and the more so the more deposits they issued in the crisis. Those assets are precisely the ones shadow banks have sold (see Figure 13). Besides, we find that those banks that purchased assets in the crisis did so at the expense of credit. This is in line with Shleifer and Vishny (2010) and Stein (2013) who discuss how market conditions shape the allocation of scarce bank capital across lending and asset purchases. Using German data, Abbassi et al. (2015) find comparable results.

2.3 Fact 3: Asset fire sales

Our illustration of asset fire sales comes from Gorton and Metrick (2011). The authors provide a snapshot of fire sales of assets that occurred due to the financial crisis that we reproduce on Figure 15 in Appendix A.3.1. We see a negative spread between higher- and lower-rate bonds with the same maturity. Aaa-rated corporate bonds would normally trade at higher prices (i.e. lower spreads) than any lower-grade bonds with the same maturity (say, Aa-rated ones), and this negative spread is an evidence of such an important amount of Aaa-rated corporate bonds sales that the spread must rise to attract buyers.

11Our model is in line with theories of financial intermediation as issuers of riskfree claims. A seminal paper is Gorton and Pennacchi (1990), and other papers include Stein (2012), DeAngelo and Stulz (2015) and Plantin (2015). As in Gennaioli et al. (2013), Krishnamurthy (2010), Caballero and Krishnamurthy (2008) or Caballero and Farhi (2016), we model households’ demand for safety as stemming from households’ risk aversion.
Other examples in the literature suggest that asset prices have deviated significantly from "fundamental values" and were sold at fire-sale prices during the crisis. Using data on insurance companies, Merrill et al. (2012) show that risk-sensitive capital requirements, together with mark-to-market accounting, can cause financial intermediaries to engage in fire sales of RBMS securities. Krishnamurthy (2008) discusses pricing relationships reflecting similar distortions on agency MBS, and notably the increasing option-adjusted spread of Ginnie Mae MBS versus the US Treasury with the same maturity. Gagnon et al. (2011) also document substantial spreads on MBS rates - well above historical norms. Such evidence of high spreads on a security which has no credit risk points to the scarcity of arbitrage capital in the marketplace and the large effects that this shortage can have on asset prices. Using micro-data on insurers’ and mutual funds’ bond holdings, Chernenko et al. (2014) finds that in order to meet their liquidity needs during the crisis, investors traded in more liquid securities such as government-guaranteed MBS. This strategy is consistent with theories of fire sales where investors follow optimal liquidation strategies: although spreads on GSE MBS were very high in the fall of 2008, those assets remained the most liquid ones in securitization markets at that time.

In our model, asset fire sales are key to understanding the relationship between traditional and shadow banks. As in Shleifer and Vishny (1992), the price of assets sold during the crisis is the price at which the best users of these assets (traditional banks in our model) can pay, given they are limited in their ability to issue deposits. Shleifer and Vishny (1997) and Gromb and Vayanos (2002) model fire sales during which mispricing occurs due to frictions on arbitrageurs’ funding capacity. Acharya et al. (2012) study interbank lending and asset sales when some banks have market power vis-a-vis other banks. Diamond and Rajan (2011) discuss liquidity risks on both sides of banks’ balance sheet, and inefficient exposure to fire sales.

3 Model setting

The model features a closed economy, with three dates \( t = 0, 1, 2 \), one production technology, two sets of agents (households and bankers) and two types of goods (consumption goods and capital goods).\(^{12}\) Date 1 includes two states \{G, B\} and date 2 three states \{GG, BG, BB\}.

3.1 Households

A unit mass of households is endowed with a large quantity of consumption goods at each date. They can consume at each date and do not discount future consumption. They have linear preferences over consumption at all dates.

At each date \( t \in \{0,1\} \), and in each state \( \omega \), household’s utility function writes as follows:

\[
U_{t,\omega} = C_{t,\omega} + E_{t,\omega} [U_{t+1}]
\]

\(^{12}\)The model is broadly inspired by Stein (2012). However, we consider two types of banks: traditional and shadow banks. It is different from the framework of Hanson et al. (2015) in that in our model traditional banks can issue debt in a crisis, and bankers endogenously choose which type of banking sector they enter.
with

\[ u_{2,\omega} = c_{2,\omega} \]  

(2)

Households are not able to invest directly in physical projects, and can only invest in financial claims issued by banks, which undertake the investment.\(^{13}\)

### 3.2 Bankers

There is a unit mass of identical bankers, who start at \( t = 0 \) with an endowment \( n \) of consumption goods. As households, they are risk neutral and indifferent between consuming at \( t = 0, 1, 2 \). Each of them chooses the probability at which they are willing to set up a traditional bank (T-bank), \( 1 - \chi^S \), and a shadow bank (S-bank), \( \chi^S \). Once allocated to one sector or the other, they invest a quantity \( n^i \in [0, n] \) \((i = \{S, T\})\) into the firm and consume whatever is left from this endowment. The funds invested in the bank constitute the own funds of the bank.

#### 3.2.1 Banks’ investment technology

Both T- and S-banks have access to a unique investment technology, whose payoffs are summarized on Figure 4.

Investing one unit of capital good in the investment technology at \( t = 0 \) yields a risky payoff \( z \in \{R, r, 0\} \) in terms of consumption goods at \( t = 2 \), in each respective state of \( \Omega_2 \equiv \{GG, BG, BB\} \). At this date, investment pays off and all capital goods is destroyed. At \( t = 1 \), information about the occurrence of date 2 states is revealed: when state G (good news) materializes (which occurs with probability \( p \)), it is known with certainty that state \( \{GG\} \) will take place at \( t = 2 \) so that investment pays off \( R \), and all uncertainty is resolved. However, when state B (bad news) materializes (which occurs with probability \( 1 - p \)), it is learnt that there is a non-zero probability of 0 output at \( t = 2 \). The probabilities that each state \( \{BG, BB\} \) materializes are \( (q, (1 - q)) \). In this case, there is an aggregate risk in the economy, that cannot be diversified away through other forms of investments.

#### 3.2.2 Bank’s choices

The timeline of the model is detailed in Figure 5.

**Time 0** At \( t = 0 \), both T- and S-banks have the ability to transform consumption goods into capital goods (or physical assets) one-for-one, in order to invest \( I^i_0 \) \((i = \{S, T\})\) units of capital goods in the long-term productive investment technology. To fund their investments, they can add to their endowment \( n^i, D^i_0 \) units of funds raised from the households. Raising funds can be done in the exclusive form of risk-free short-term debt, issued on a competitive market.

This assumption captures what we see as one fundamental role of banks: their ability to act as safety and liquidity providers to the households at all times. The key assumption here is that banks have to

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\(^{13}\) As in Stein (2012) we abstract from any agency problem between the intermediary and the firm manager and assume that the bank has all the bargaining power in the banking-firm relationship, thereby enabling it to extract all the profit from the investment and leaving the firm with no profit in expectation.
Figure 4: Investments payoffs (states in bold font)

Figure 5: Timeline of the model
finance themselves with risk-free financial contracts. Our results do not rely on the specific form of these contracts.  

**Time 1**  At $t = 1$, T- and S-banks can’t either transform consumption goods into capital goods, or transform capital goods into consumption goods. They can however trade capital goods for consumption goods in a competitive secondary market, where all banks participate. In each state $\omega_1 \in \{B, G\}$, they purchase $I_{1,\omega_1}^i$ ($i = \{S, T\}$) units of capital goods at a market price $p_{1,B}q_r$ at $t = 1$ in state B and $p_{1,G}R$ at $t = 1$ in state G. These capital goods are then reinvested in the same technology.  

Banks can also sell on the market a share $(1 - \alpha_{1,\omega_1}^i)$ of their stock of physical assets. We assume liquidation costs for banks such that a share $\varepsilon \in [0, 1]$ of assets sold is destroyed. This cost assumption is meant to capture the forsaken returns from liquidating illiquid projects.  

Finally, banks have the ability to raise additional funds from households, $D_{1,\omega_1}^i$, by issuing risk-less short-term debt, either to roll previously issued debt over, or to finance purchases. The debt issued at $t = 0$ must be paid back, such that $r_0 D_{0}^i$ units of consumption goods must be provided to the debt-holders, where $r_0$ is the interest rate on the debt issued at $t = 0$, pinned down on the market.  

**Time 2**  At $t = 2$, the investment pays off in terms of consumption goods, and all capital goods are destroyed. T and S banks repay their date 1 debt-holders by providing them with $r_{1,\omega_1} D_{1,\omega_1}^i$ (with $\omega_1 \in \{B, G\}$) units of consumption goods, where $r_{1,\omega_1}$ is the interest rate on the debt issued at $t = 1$ in state $\omega_1 \in \{B, G\}$. It is important to note that in each date and state, banks are subject to limited liability constraints.

### 3.2.3 Differences between shadow and traditional banks

We distinguish traditional from shadow banking by making the following assumptions.

**Assumption 1** (Differences). Traditional and shadow banks differ in two ways:

1. **T-banks have access to a guarantee fund at $t = 1$ in state B.** This enables them to issue risk-less claims that promise to pay up to an amount $k > 0$.  

2. **T-banks have to cope with higher operating costs**: At $t = 2$, T-banks only get a fraction $\delta \in [0, 1]$ of the payoff generated by their investments.

T-banks benefit from an advantage over S-banks in $t = 1$ in state B they have access to a government guarantee which enables them to issue risk-less short-term debt from $t = 1$ in state B to $t = 2$, even if the productive technology is facing a risk of zero output. In our setting, it takes the form of a fairly-priced guarantee fund owned by the state: to get one unit of consumption good at $t = 2$ in state BB,
bankers have to provide \( \frac{1-k}{q} \) units of consumption goods to the fund at \( t = 2 \) in state BG such that the government is making no profit in expectation from setting this fund up, and provides no subsidy to the T-banks through this fund. We assume that the maximum guarantee each T-bank can benefit from is set to \( k \), which is a structural parameter of our economy. One interpretation for this is that the government’s ability to enforce payments made by T-banks to the fund at \( t = 1 \) in state G is limited, for instance due to limited fiscal capacity. Another interpretation for this parameter could be a reduced form for informational friction which prevents T-banks from taking too much debt at \( t = 1 \) in state B\(^17\).

Note that, T-banks being subject to limited liability constraint at all times and states, \( k \) is not the only determinant of T-banks debt level, as will be emphasized below.

In return to this advantage, T-banks face regulatory costs, which take the form of a reduced payoff on their investment at \( t = 2 \) in each state. This cost captures a wide variety of costs associated to higher regulations imposed to the traditional banking sector: taxes to finance the regulating entities, the general functioning costs of the guarantee fund, costs to generate regulatory information, etc.\(^18\)

### 3.3 Equilibrium definition

In this setting, we formally define an equilibrium as follows.

**Definition 1 (Equilibrium).** An equilibrium is defined by a set \( \{ \chi^S, n^S, n^T, v^S, v^T, p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B} \} \)

with \( v^i = \{ I^i_0, D^i_0, I^i_{1,G}, I^i_{1,B}, D^i_{1,G}, D^i_{1,B}, \alpha^i_{1,G}, \alpha^i_{1,B} \} \)

for \( i \in \{ S, T \} \) such that

1. \( v^T \) maximizes traditional bank’s date 0 value function \( V_0^T (p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B}, n^T) \).
2. \( v^S \) maximizes shadow bank’s date 0 value function \( V_0^S (p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B}, n^S) \).
3. \( n^i \) maximizes the expected payoff of a banker’s allocated in type i-bank (\( i \in \{ S, T \} \)) \( V_0^{i,B} (p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B}) \).
4. \( \chi^S \) is a solution to the allocation problem:

\[
\max_{\chi^S \in [0;1]} \chi^S V_0^{S,B} (p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B}) + (1 - \chi^S) V_0^{T,B} (p_{1,G}, p_{1,B}, r_0, r_{1,G}, r_{1,B}).
\]

5. Markets for short-term debt clear at time 0, and 1, in states G and B for respective interest rates \( \{ r_0, r_{1,G}, r_{1,B} \} \).
6. Capital goods market clear at time 1 in states G and B at respective prices \( \{ p_{1,G} q^G, p_{1,B} q^B \} \).

We now turn to the analysis of this equilibrium, detail the banks and bankers’ problems and value functions, as well as the market clearing conditions, and develop the solutions.

\(^{17}\)For such an interpretation, we will see that it is not restrictive not to impose the same constraint on S-bank: the existence of a risk of zero-payoff at \( t = 2 \) in state BB prevents S-banks to issue any type of risk-less debt at \( t = 1 \) in state B making such a constraint superfluous.

\(^{18}\)From a positive perspective, it can also be interpreted more broadly as a series of costs associated to the T-banks specificities in terms of business model: they have higher operating costs, employ more workers, provide more services to their customers etc.
4 Model analysis

4.1 Assumptions

Assumptions on expected returns In Assumption 2 we make two restrictions about the investment technology.

**Assumption 2 (Returns).** At $t = 0$, investing is efficient for both $T$- and $S$-banks:

$$
\delta (pR + (1 - p)qr) > 1
$$

At $t = 1$ in state B, expected returns are lower than one:

$$
qr < 1
$$

Condition (3) ensures that as of $t = 0$, investing is efficient. This implies that each banker invests her full endowment in the bank it sets up, be it a S-bank ($n^S = n$) or a T-bank ($n^T = n$), and each bank invests all her own funds in their time 0 investment technology. Condition (4) reflects the fact that in a crisis, asset returns are lower than in good state.

Assumptions on traditional bank’s parameters We make two additional assumptions on the size of the guarantee traditional banks can benefit from, $k$, as well as its associated cost, $\delta$, which helps us focus on the main cases of interest.

**Assumption 3 (Banks’ parameters).** The cost associated to traditional banking activity is low enough to prevent asset sales at $t = 1$ in state G:

$$
\delta > 1 - \varepsilon
$$

The guarantee T-banks can benefit from on his time 1, B claims is the binding constraint for time 0, T-banks debt issuance:

$$
k < k^* = \frac{\delta qr}{1 - \delta qr} n
$$

Condition (5) enables us to rule out asset transfers at $t = 1$ in state G between the two types of intermediaries: it will always be optimal for any type of bank to choose to continue their time 0 investment, at $t = 1$ in state G rather than providing assets on the market for capital goods. This assumption could be relaxed at little cost. It is however convenient in keeping the analysis focused on the interaction between the two types of banks in the bad information state, time 1. Relaxing this assumption generates asset transfers from T-banks to S-banks in the good information state: indeed, if $\delta < 1 - \varepsilon$, it is more valuable for a T-bank to sell capital goods on the market to S-banks who value it more\(^{19}\), and incur the illiquidity cost $\varepsilon$, than continuing its investment and incur the regulatory cost $1 - \delta$ on its time 2 payoff.

\(^{19}\)Each unit of capital good generates a return $R$ for a S-bank in date 2, GG instead of $\delta R$ for T-banks, and none of them discount future payoffs.
The second assumption (6) will imply that the maximum amount of risk-less debt issued at \( t = 0 \) that can be rolled-over at \( t = 1 \) in state B by a T-bank will be constrained by the size of the guarantee fund T-banks have access to at \( t = 1 \) in state B (hence \( k \)), and not the limited liability constraint of time 2, BG the bank has to comply with when financing this guarantee. This ensures that T-banks will be able to issue an amount \( k \) of risk-less debt both at \( t = 0 \) and at \( t = 1 \) in all states.

**Technical assumption** Finally, we set the following assumption, in order to ensure coexistence of the two types of institutions.

**Assumption 4 (Assumption for coexistence).** We assume that the cost of regulation is high enough such that both S and T-banks can exist in equilibrium

\[
\delta < \frac{p}{\varepsilon (p(R - 1) + (1 - p)qr) + p}
\]

(7)

We discuss later the fact that condition (7) is a necessary condition to find a range of asset prices at \( t = 1 \) in state B such that both T and S-banks are willing to trade assets, given condition (6).\(^{20}\)

### 4.2 Equilibrium implications of the assumptions

We now turn to the resolution of the equilibria of the game. In order to reduce the dimensionality of the exercise and ease further exposition we start by detailing some equilibrium conditions implied by our assumptions. We consider equilibrium conditions associated to the debt market, the asset market at \( t = 1 \) in state G, and discuss their implications.

#### 4.2.1 Debt market clearing conditions

Households are endowed with a large amount of consumption goods at each date, and the ability to generate safe short-term claims in the system being limited,\(^{21}\) the real rates on short term debt issued by any type of bank is pinned down in equilibrium by the household’s utility function. With a linear utility function and no time discounting, this rate is set to 1 in equilibrium.

In equilibrium:

\[
r_0 = r_{1,B} = r_{1,G} = 1
\]

It is important to note that, the households being endowed with a large amount of consumption goods in each date and state, the limiting factor in banks’ ability to expand will always be their ability to generate safe short-term claims. We prevent in this way mispricing of the debt instruments to play any role in the mispricing of assets that could occur when assets flow from one type of bank to the other.

\(^{20}\)It is important to note that the five parametric restrictions generated by these assumptions are not mutually exclusive.

\(^{21}\)It is indeed constrained by the guarantee that can be obtained from the government, which implies that the maximum amount of short term risk-less claims that can be issued at \( t = 0 \) or \( t = 1 \) cannot exceed \( k(1 - X^S) \).
4.2.2 Time 1, G asset market clearing condition

at \( t = 1 \) in state G, T and S-banks must repay their date 0 debt-holders \( r_0 D_0^i = D_0^i \) (\( i = S, T \)). They can choose to issue short term risk-less debt, sell part of their capital goods to other banks on the asset market, and purchase capital goods sold by others. In this time and state, all uncertainty is resolved, such that both S and T-banks are able to generate short term safe claims \( D_{1,G}^i \) (\( i = S, T \)) to roll their funding over as long as \( D_{1,G}^T \leq \delta R_{0}^T \) and \( D_{1,G}^S \leq R_{0}^S \) (due to limited liability constraint at \( t = 2 \) in state GG).

This strategy is a dominating one for S-banks as well as T-banks (due to assumption (5)), as it spares them the destruction of capital goods \( \epsilon \) associated to sale of capital goods: if T-banks were to sell their assets on the market, they could get \( (1 - \epsilon) R_{0}^T \) as S-banks would be willing to purchase the assets at fair price \( R \), by issuing risk-less short term debt to finance this purchase. This being lower than \( \delta R_{0}^T \), the value of keeping the assets in place, T-banks will always choose to hold on their time 0 investments, as would S-banks do.

Finally, in equilibrium, no capital goods are supplied on the market, and no trade takes place. Without loss of generality, we simplify S and T-banks programs exposition by presenting and solving them as if no asset market existed at \( t = 1 \) in state G, and their only option were to roll their funding over, as these two problems are equivalent in equilibrium. We can now turn to the exposition of these modified banks’ programs.

4.3 Shadow banks’ program

We solve a S-bank’s program given it has no access to the asset market at \( t = 1 \) in state G.

Date 1, state G. In such a program, at \( t = 1 \) in state G, S-banks can only roll their funding over. The value function of a S-bank at \( t = 1 \) in state G, with a date 0 investment level \( I_{0}^S \), a debt level \( D_{0}^S \) writes, in case of no-default, as follows:

\[
V_{1,G}^{S,ND} \left( I_{0}^S, D_{0}^S \right) = R_{0}^S - D_{0}^S
\]

and the S-bank doesn’t default if and only if \( R_{0}^S - D_{0}^S \geq 0 \).

At \( t = 1 \) in state G, if the S-bank has a higher debt level inherited from \( t = 0 \), it would have to default on it and would not be able to repay its creditors.

As S and T-banks can only issue risk-less short term debt in each date and state, such an event would not occur in equilibrium. We take the convention to set, in these cases, the value function to \(-\infty\). S-banks value function at \( t = 1 \) in state G writes:

\[
V_{1,G}^{S} \left( I_{0}^S, D_{0}^S \right) = \begin{cases} 
R_{0}^S - D_{0}^S & \text{if } D_{0}^S \leq R_{0}^S \\
-\infty & \text{otherwise}
\end{cases}
\]

Date 1, state B. At \( t = 1 \), in state B, S-banks choose how much (if any) funding to raise, how much capital goods to sell on the market, and how much capital goods to purchase on the market, for making additional investments in their technology. They also have to repay their date 0 debt-holders.
At $t = 1$ in state $B$ there is a non-zero probability for the investment technology to return a zero payoff at $t = 2$. Therefore, in the absence of any guarantee on the claims they issue, S-banks are not able to issue any risk-less claim at $t = 1$ in state $B$. The only way for S-banks to repay households is to sell capital goods on the market and channel the proceeds of this sale to households so as to meet their obligations. Shadow banks can also choose to interrupt a higher fraction of their date 0 investment, to sell more capital goods on the market, in order either to consume or purchase capital goods from other banks at $t = 1$ in state $B$.

Remark that S-banks can only invest at $t = 1$ in state $B$ by interrupting previous investments made at $t = 0$. It can never be optimal for a S-bank to interrupt their time 0 investments, which induces an early liquidation cost $\epsilon$, in order to sell capital goods on the market at a price $p_{1,B} qr$ by unit of capital goods, purchase new capital goods at the same price and reinvest it in the same investment technology. Such a strategy is indeed strictly dominated by the one that consists in keeping the investments without liquidation.

We therefore write the shadow bank’s value function at $t = 1$ in state $B$ in the case of no default, as

$$V_{t=1,B}^{SD,ND}(I_{0}^{S}, D_{0}^{S}; p_{1,B}) = \max_{\alpha_{t,B}^{S} \in [0;1]} \alpha_{t,B}^{S} qrI_{0}^{S} + (1 - \alpha_{t,B}^{S})(1 - \epsilon)p_{1,B} qrI_{0}^{S} - D_{0}^{S}$$

s.t. $(1 - \alpha_{t,B}^{S})(1 - \epsilon)p_{1,B} qrI_{0}^{S} \geq D_{0}^{S}$

where $I_{0}^{S}$ is the investment level of the S-bank at $t = 0$, $D_{0}^{S}$ the investment level of the S-bank at $t = 0$, $p_{1,B} qr$ the market price of one unit of capital good, $\alpha_{t,B}^{S}$ is the share of investment made at $t = 0$ that the S-bank is willing to pursue at $t = 1$ in state $B$, and the S-bank is subject to a limited liability constraint, as well as positivity and no short-sale constraints.

Keeping the same convention, we set the value function in case of default to $-\infty$.

**Lemma 1** (S-banks time 1). At $t = 1$, in state $B$, S-banks don’t default if and only if $D_{0}^{S} \leq (1 - \epsilon)p_{1,B} qrI_{0}^{S}$.

If $p_{1,B} > 0$, their value function writes

$$V_{t=1,B}^{S}(I_{0}^{S}, D_{0}^{S}; p_{1,B}) = \begin{cases} (1 - \epsilon)p_{1,B} qrI_{0}^{S} - D_{0}^{S} \max \left( \frac{1}{(1 - \epsilon)p_{1,B}}; 1 \right) & \text{if } D_{0}^{S} \leq (1 - \epsilon)p_{1,B} qrI_{0}^{S} \\ -\infty & \text{otherwise} \end{cases}$$

If $p_{1,B} = 0$, their value function writes

$$V_{t=1,B}^{S}(I_{0}^{S}, D_{0}^{S}; p_{1,B}) = \begin{cases} qrI_{0}^{S} & \text{if } D_{0}^{S} = 0 \\ -\infty & \text{otherwise} \end{cases}$$

**Date 0**  At $t = 0$, Shadow banks will always choose levels of debt $D_{0}^{S}$ and investment $I_{0}^{S}$ consistent with an absence of default on their debt at $t = 1$, as they are only able to raise funding in such a way. They also have to face a funding constraint: their time 0 investment is funded with the banker’s endowment and the debt raised from households at $t = 0$. 22

22Thanks to condition (3), we know that bankers will always choose to invest all their endowment into the bank they set up.
For $p_{1,B} > 0^{23}$, the date 0 value function of a S-bank, with own funds $n^S$ writes:

$$V_0^S(p_{1,B}, n^S) = \max_{D_0^S, t_0^S \geq 0} [(1 - p) (1 - \epsilon) p_{1,B} q r t_0^S - D_0^S] \max \left( \frac{1}{(1 - \epsilon) p_{1,B}}; 1 \right)$$

$$+ p \left( R t_0^S - D_0^S \right) + (D_0^S + n^S - t_0^S)]$$

s.t. $D_0^S + n^S \geq t_0^S$

$D_0^S \leq (1 - \epsilon) p_{1,B} q r t_0^S$

$D_0^S \leq R t_0^S$

Denoting $p_1^S = \frac{1}{(1 - \epsilon) (q r + \frac{p_{1,B} qr}{1 - p})} < \frac{1}{1 - \epsilon}$, we obtain the following proposition.

**Proposition 1** (S-bank program). At $t = 0$, S-banks take the following decisions.

1. If $0 \leq p_{1,B} < p_1^S$, $D_0^S = 0$, $t_0^S = n^S$. Shadow banks do not issue short-term claims at $t = 0$, i.e. they do not lever themselves. In this case, $V_0^S(p_{1,B}, n^S) = (1 - p) q r n^S + p R n^S$

2. If $p_{1,B} = p_1^S$, any $D_0^S \in \left[ 0; \frac{1 - \epsilon}{1 - (1 - \epsilon) p_{1,B} q r} n^S \right]$ is an equilibrium solution, and $t_0^S = n^S + D_0^S$. Shadow banks sell a fraction $\frac{D_0^S}{1 - \epsilon} p_{1,B} q r (D_0^S + n^S)$ of their capital goods at $t = 1$, B, so as to repay their date-0 creditors. In this case, $V_0^S(p_{1,B}, n^S) = (1 - p) q r n^S + p R n^S$

3. If $p_1^S < p_{1,B} < \frac{1}{1 - \epsilon} q r$, $D_0^S = \frac{(1 - \epsilon) p_{1,B} q r}{1 - (1 - \epsilon) p_{1,B} q r} n^S$, $t_0^S = n^S + D_0^S$. Shadow banks sell all their capital goods at $t = 1$ in state B, so as to repay their date-0 creditors. In this case, $V_0^S(p_{1,B}, n^S) = p (\frac{R - (1 - \epsilon) p_{1,B} q r}{1 - (1 - \epsilon) p_{1,B} q r}) n^S$

4. If $p_{1,B} \geq \frac{1}{1 - \epsilon} q r$, $D_0^S = +\infty$, $t_0^S = +\infty$ and $V_0^S(p_{1,B}, n^S) = +\infty$

**Proof.** See appendix. □

Although S-banks do not have access to the guarantee fund, they can issue risk-free debt at $t = 0$ insofar as they are backed by the liquidation value of the fraction $(1 - \alpha_{1,B}^S)$ of their existing investment they sell at $t = 1$ in state B. It is S-banks’ ability to pull the plug in the crisis that enables them to issue safe short-term debt at $t = 0$. When liquidating at $t = 1$ in state B, proceeds from selling capital goods are $(1 - \alpha_{1,B}^S)(1 - \epsilon)p_{1,B} q r t_0^S$ where $p_{1,B} q r$ is the price of a unit of capital good in the secondary market at $t = 1$ in state B. The proceeds of this sale depend on T-banks’ ability to purchase capital goods in the crisis, which itself relies on the guarantee fund these latter can access.

Indirectly, S-banks therefore rely on the guarantee fund via T-banks, thereby granting T-banks the ability to play the role of government support intermediary. They are able to purchase guarantee from the government at fair price and sell it to S-banks at a potential premium.

---

If $p_{1,B} = 0$ this reduces to

$$V_0^S(0, n^S) = \max (1 - p) (q r t_0^S) + p (R t_0^S)$$

s.t. $n^S \geq t_0^S$

$t_0^S \geq 0$
4.4 Traditional banks’ program

We now turn to T-banks’ program, which we expose and solve again by backward induction.

As for S-banks, and for the clarity of exposition, we expose and solve, without loss of generality, the modified problem, of a T-bank with no access to capital goods market at $t = 1$ in state $G$, and whose only option is to roll its funding over.

**Date 1, state $G$.** As for S-banks, at $t = 1$ in state $G$, T-banks roll their funding over. With the same convention on value function in case of default, T-banks value function at $t = 1$ in state $G$ writes:

$$V_{1,G}^{T} (I_{T}^{0}, D_{T}^{0}) = \begin{cases} \delta RI_{T}^{0} - D_{T}^{0} & \text{if } D_{T}^{0} \leq \delta RI_{T}^{0} \\ -\infty & \text{otherwise} \end{cases}$$

**Date 1, state $B$.** In contrast to S-banks, T-banks are able to issue safe claims at $t = 1$ in state $B$ because they can access a guarantee fund that makes these claims safe despite a non-zero probability of a zero output at $t = 2$. The possibility of a zero output at $t = 2$ therefore does not deter T-banks from issuing claims to households at $t = 1$ in state $B$, but T-banks are subject to two constraints in this respect. They have (i) to fairly pay for the guarantee on their short-term debt, while complying with their time 2 limited liability conditions and (ii) to comply with their debt constraint $k$.

Constraint (i) puts an upper bound on the amount of risk-less debt that can be repayed at $t = 2$ in states $BB$ and $BG$ (or similarly issued at $t = 1$ in state $B$): this amount cannot exceed the expected payoff of the productive investment at $t = 2$.

Indeed, the guarantee fund does not provide any subsidy to the T-bank: the T-bank must repay its date 1, B debt $D_{1,B}^{T}$ at $t = 2$, either by reimbursing the debt $D_{1,B}^{T}$ directly (at $t = 2$ in state $BG$), or by fairly financing its time 2, BB guarantee at $t = 2$ in state $BG$, hence providing the guarantee fund with $\frac{1-q}{q} D_{1,B}^{T}$ at $t = 2$ in state $BG$, such that the expected net payment made to the fund, from a time 1, B perspective is:

$$\frac{1-q}{q} D_{1,B}^{T} + (1-q)(-D_{1,B}^{T}) = 0$$

For a T-bank, with a date 0 investment level $I_{T}^{0}$, who chooses to keep a share $\alpha_{1,B}^{T}$ on its balance sheet, and purchases $I_{1,B}^{T}$ units of capital goods at $t = 1$ in state $B$, the limited liability constraint at $t = 2$ in state $BG$ rewrites (with $D_{1,B}^{T}$ the amount of risk-free short term debt issued at $t = 1$ in state $B$):

$$\frac{1-q}{q} D_{1,B}^{T} + D_{1,B}^{T} \leq \delta r \left( I_{1,B}^{T} + \alpha_{1,B}^{T} I_{0}^{T} \right)$$

Or equivalently

$$D_{1,B}^{T} \leq \delta qr \left( I_{1,B}^{T} + \alpha_{1,B}^{T} I_{0}^{T} \right)$$

Constraint (ii) writes as follows

$$D_{1,B}^{T} \leq k$$

In addition to these constraints, the bank is facing a limited liability constraint at $t = 1$ in state $B$ and must repay its date 0 debt-holders and finance its date 1, $T$ purchase either by raising new debt, or
by selling part of its capital goods on the market. It also faces the same no short-sale constraint, and positivity constraints as the S-banks.

In the no-default case, the value function of a T-bank at $t = 1$ in state B, writes

$$V_{1,B}^{T,ND} (I_0^T, D_0^T, p_{1,B}) = \max_{(d_{1,B}, d'_{1,B}, t_{1,B}) \in \{0\} \times \{0, 1\} \times \mathbb{R}_+} \left\{ \left( -p_{1,B} + s_{1,B} + s_{1,B} + (1 - s_{1,B})p_{1,B} + s_{1,B}q_{1,B} (1 - \varepsilon) - D_0^T \right) \right. \right.$$

$$\left. s_{1,t} (1 - p_{1,B}) q_{1,B} + s_{1,B}q_{1,B} (1 - \varepsilon) + D_{1,B} \geq D_0^T + p_{1,B} q_{1,B} (1 - \varepsilon) \right. \right.$$

$$\left. d_{1,B} \leq \alpha \left( + s_{1,B}q_{1,B} (1 - \varepsilon) \right) \right\}$$

We keep the convention of setting $V_{1,B}^{T,ND} (I_0^T, D_0^T, p_{1,B}) = -\infty$ if the bank defaults on its time 0 debt-holders.

As shown in the appendix, the T-bank does not default on its debt if and only if $D_0^T \leq D_0^T (I_0^T, p_{1,B})$, where $D_0^T (I_0^T, p_{1,B})$ is given in the appendix. The value function of a T-bank at $t = 1$ in state B is given in Proposition 2.

**Proposition 2 (Time 1, T-banks).** For any $I_0^T \geq 0$, $D_0^T \geq 0$ and $p_{1,B} > 0$,

$$V_{1,B}^{T} (I_0^T, D_0^T, p_{1,B}) = \begin{cases} V_{1,B}^{T,ND} (I_0^T, D_0^T, p_{1,B}) & \text{if } D_0^T \leq D_0^T (I_0^T, p_{1,B}) \\ -\infty & \text{otherwise} \end{cases}$$

with

$$V_{1,B}^{T,ND} (I_0^T, D_0^T, p_{1,B}) = \left( \frac{\delta - p_{1,B}}{p_{1,B}} + (k - D_0^T) + \left( k - D_0^T \right) - \frac{\delta - p_{1,B} (1 - \epsilon)}{p_{1,B} (1 - \epsilon)} \right)$$

$$+ (p_{1,B} (1 - \epsilon) - \delta) + q_{1,B} D_0^T + q_{1,B} \delta q_{1,B} D_0^T - D_0^T$$

Moreover, if $p_{1,B} = 0$, $V_{1,B}^{T,ND} (I_0^T, D_0^T, p_{1,B}) = +\infty$

**Proof.** See appendix

The intuition goes as follows. If the debt raised at $t = 0$ is higher than the maximum guarantee traditional banks can benefit on the time 1, B claims they issue $D_0^T > k$, the T-bank is not able to roll over all its funding. According to the price level, it will either choose to roll over as much of its previous debt as possible, hence $k$, (when the price is low), and sell just a high enough share of its assets, to cover for the remaining debt that must be repaid, or might choose not to roll over any debt, and sell capital goods on the market if the price gets high enough.

However, if the debt raised at $t = 0$ can be fully repaid by raising short term debt $t = 1$ in state B, the T-bank shall choose to roll all its funding over for low level of prices. Moreover, if prices are low enough, T-banks will choose to bind their $k$-constraint (set $D_{1,B}^T \leq k$) by raising additional debt such as to purchase under-priced capital goods. For higher level of prices, T-banks would choose not to invest, and only raise additional debt in order to repay its date 0 claim-holders, as well as potentially increase its time 1 consumption. Finally, if the price is high enough, T-banks might also be willing to sell all its capital goods on the market.

If the date 0 debt level is higher than the payoff that can be obtained with these strategies, the T-bank would not be able to repay all its date 0 debt-holders, while still complying with the different constraints.
it has to face. It would then only be able to repay less than the debt face value, which would lead to a default. In this case, the convention we chose is to put the value function to $-\infty$.

From a date 0 perspective, as T-banks can only finance themselves through risk-less short term debt, such a high level of debt cannot be chosen in equilibrium.

**Date** $t = 0$. At $t = 0$, T-banks choose how much funds to raise, and how much consumption goods to transform into capital goods. As for S-banks, T-banks have to choose a debt level such that no-default occurs. Moreover, the same funding constraint applies.

T-banks value function at $t = 0$ writes:

$$V_0^T(\bar{p}_{1,B}, n^T) = \max_{(D_0^T, I_0^T) \in \mathbb{R}_+^2} p \left( \delta R I_0^T - D_0^T \right) + (1 - p)V_{1,B}^T \left( D_0^T, I_0^T, \bar{p}_{1,B} \right) + (D_0^T + n^T - I_0^T)$$

Moreover, if $\bar{p}_{1,B} = \frac{\delta}{\delta \rho + p \delta R - q \rho} < \delta$, and $\bar{p}_{1,L} = \frac{\bar{p}_{1,L}}{\delta \rho} < \frac{\delta}{\delta \rho}$, T-banks' program yields the following proposition.

**Proposition 3 (T-banks' program).** Under condition (6), T-bank's program solves as follows

1. If $0 < p_{1,B} < p_{1,L}^T$, $D_0^T = 0, I_0^T = n, V_0^T = p \left( \delta R(n + k) - k \right) + (1 - p) \left( \delta R(n + k) - k \right)$
2. If $p_{1,B} = p_{1,L}^T$ any $D_0^T \in [0, k]$, $I_0^T = n + D_0^T$ is an equilibrium solution and $V_0^T = p \left( \delta R(n + k) - k \right) + (1 - p) \left( \delta R(n + k) - k \right)$
3. If $p_{1,B} < p_{1,L}^T < p_{1,L}, D_0^T = k, I_0^T = k + n$ and $V_0^T = p \left( \delta R(n + k) - k \right) + (1 - p) \left( \delta R(n + k) - k \right)$
4. If $p_{1,B} = p_{1,L}^T$ any $D_0^T \in [k, \frac{k \left( \frac{1 - \delta}{\delta \rho + \delta R - q \rho} \right) + (1 - \delta) \delta R \bar{p}_{1,B} n}{1 - \frac{1 - \delta}{\delta \rho + \delta R - q \rho}}]$, is an equilibrium solution, $I_0^T = n + D_0^T$ and $V_0^T = p \left( \delta R(n + k) - k \right) + (1 - p) \left( \delta R(n + k) - k \right)$
5. If $p_{1,B} \leq \frac{\delta}{\delta \rho}, D_0^T = \frac{k \left( \frac{1 - \delta}{\delta \rho + \delta R - q \rho} \right) + (1 - \delta) \delta R \bar{p}_{1,B} n}{1 - \frac{1 - \delta}{\delta \rho + \delta R - q \rho}}, I_0^T = n + D_0^T + V_0^T = p \left( \delta R(D_0 + n) - D_0 \right)$
6. If $\frac{\delta}{\delta \rho} \leq p_{1,B} < \frac{1}{\delta \rho + \delta R - q \rho}, D_0^T = \frac{p_{1,B}(1 - \delta) \delta R \bar{p}_{1,B} n}{1 - \frac{1 - \delta}{\delta \rho + \delta R - q \rho}}, I_0^T = D_0^T + n$, and $V_0^T = p \left( \delta R(D_0 + n) - D_0 \right)$
7. If $p_{1,B} \geq \frac{1}{\delta \rho + \delta R - q \rho}, D_0^T = +\infty, I_0^T = +\infty$ and $V_0^T = +\infty$

Moreover, if $p_{1,B} = 0, V_0^T = +\infty$

Depending on the price of capital goods on the secondary market at $t = 1$ in state B, T-banks choose how much short-term debt to issue at $t = 0$ to invest in positive NPV projects, versus how much buffer to keep to purchase capital goods from S-banks at $t = 1$ in state B. Although the guarantee fund enables T-banks to issue short-term debt at $t = 1$ in state B, they have to trade-off between those two investment opportunities because their issuance of short-term debt is limited by the size of the support.
For low level of prices, the return T-banks get on purchasing capital goods on the market overcomes the one of investing in the positive NPV projects they are faced with at $t = 0$. They prefer not issuing short term debt at $t = 0$ to keep slack in order to purchase goods at $t = 1$ in state B. When prices get higher, they are better off investing even more at $t = 0$, and selling part of his capital goods in the market when a crisis hits. As shall be seen shortly, this would not occur in equilibrium.

One implication which is worth emphasizing is that the size of the support the T-bank can benefit from the government at $t = 1$ in state B has an impact on its time 0 maximum debt level. Indeed, thanks to the guarantee fund, T-banks get the ability to raise risk-free debt at $t = 0$ up to an amount $k$ under condition (6), without the need to delever at $t = 1$ in state B. The T-bank is then able to issue up to $k$ short term risk-less deposits whatever price prevails on the market at $t = 1$ in state B, generating a positive spillover from guarantee at $t = 1$ in state B to $t = 0$.

### 4.5 Banker’s endowment allocation

Finally, given $V^i_0 (p_{1,i}, n^i)$, bankers who choose to set up an i-bank ($i \in \{S; T\}$), allocate their initial endowment $n$ between a part $n^i$ they invest as bank’s own funds, and the remaining part $(n - n^i)$ they consume.

When investing $n^i \in [0; n]$ units of their endowment into an i-bank, they obtain the residual payoffs of the bank. This provides them with an expected utility $V^i_0 (p_{1,i}, n^i)$ from a date 0 perspective. Their problem writes as follows

$$V^{i,B}_0 (p_{1,B}) = \max_{n^i \in [0; n]} (n - n^i) + V^i_0 (p_{1,B}, n^i)$$

Then

1. If $p_{1,B} < \frac{1}{(1 - \epsilon) qr}$, they choose $n^i = n$. Indeed, each unit of funds a banker allocates to the i-bank can at least be transformed into capital goods and invested into the investment technology the bank has access to. The expected payoff generated by such a strategy is at least $\delta (p R + (1 - p) qr)$, which provides more utility to the banker than immediate consumption of the funds (assumption 3)

2. If $p_{1,B} \geq \frac{1}{(1 - \epsilon) qr}$ the banker is indifferent between the different levels of endowment allocation.

In any case,

$$V^{i,B}_0 (p_{1,B}) = V^i_0 (p_{1,B}, n)$$

### 4.6 Capital goods market clearing

In this section, we derive the market-clearing conditions for the capital goods market at $t = 1$ in state B, taking the shares $\chi^S (1 - \chi^S)$ of S-banks (T-banks) as given. Let us define an equilibrium on the secondary market for capital goods at $t = 1$ in state B.\(^{24}\)

\(^{24}\)The bank has always the choice to roll its time 0 debt over in this case.
Definition 2 (Market equilibrium definition). A market equilibrium at $t = 1$ in state $B$ is defined by

1. A quantity $S(p_{1,B})$ of capital goods supplied
2. A quantity $D(p_{1,B})$ of capital goods demanded
3. A price $p_{1,B}$ such that $D(p_{1,B}) = S(p_{1,B})$

We have the following proposition

Proposition 4 (Supply and demand). In date 1, B, with $\chi^S \in [0; 1]$ denoting the share of S-banks and $(1 - \chi^S)$ that of T-banks, the aggregate demand for capital goods writes:

\[
D(p_{1,B}) = \begin{cases} 
+\infty & \text{if } p_{1,B} = 0 \\
\frac{k}{p_{1,B}q_T} (1 - \chi^S) & \text{if } 0 < p_{1,B} < p^T_{1,L} \\
0 & \text{if } p_{1,B} = p^T_{1,L} \\
\frac{k}{p_{1,B}q_T} (1 - \chi^S) & \text{if } p_{1,B} > p^T_{1,L}
\end{cases}
\]

and the aggregate supply of capital goods writes:

\[
S(p_{1,B}) = \begin{cases} 
0 & \text{if } 0 \leq p_{1,B} < p^S_1 \\
\frac{n(1-\epsilon)}{1-(1-\epsilon)p_{1,B}q_T} \chi^S & \text{if } p_{1,B} = p^S_1 \\
\frac{n(1-\epsilon)}{1-(1-\epsilon)p_{1,B}q_T} \chi^S & \text{if } p^S_1 < p_{1,B} < p^T_{1,H} \\
\frac{n(1-\epsilon)}{1-(1-\epsilon)p_{1,B}q_T} \chi^S + \frac{(1-\epsilon)(-\frac{1}{2} + qr(n+k))}{1-p_{1,B}(1-\epsilon)q_T} (1 - \chi^S) & \text{if } p_{1,B} = p^T_{1,H}
\end{cases}
\]

If $p_{1,B} > p^T_{1,H}$, the aggregate supply of capital goods is strictly larger than the aggregate demand such that the market cannot clear.

Proof. See appendix

The supply and demand for capital goods are illustrated in Figure 6 and follow from our previous analysis: when the price is low, there is a high demand for capital goods by T-banks. They prefer keeping slack at $t = 1$, to take the chance of buying underpriced assets at $t = 1$ in state B. When prices increase, they are better off issuing debt and investing at $t = 0$.

When the price is low shadow banks are not willing to lever themselves at $t = 0$, because the price they would get by selling assets on the capital goods market is too low, which makes the cost of time 0 debt too high for them, the intermediation price of government guarantee being too high. When prices increase, this reduces the cost of making their short-term debt safe, hence the cost of levering. This tradeoff is akin to the one emphasized in Stein (2012).
One notices that \( S(p_{1,B}) > 0 \) when \( p_{1,B} > p_{1,T}^T \). Indeed, for such prices, both \( S \) and \( T \)-banks are willing to sell capital goods, which induces a positive supply whatever the allocation of intermediaries between the two types of banks. This restricts the set of prices that can prevail in equilibrium to \([0; p_{1,T}^T]\).

Our technical assumption (7) ensures that \( p_{S}^T < p_{1,L} < p_{1,T}^T \). According to the share of \( S \)-banks, different equilibria on the capital goods market can prevail.

**Proposition 5 (Market equilibrium).** In equilibrium, we have

1. Either \( \chi_S = 0, D = S = 0, \) and \( p_{1,L} \leq p_{1,B} \leq p_{1,T}^T \). No assets are traded.

2. Or \( \chi_S \in (0;1), D = S, \) and

   (a) Either \( \frac{k}{k} \leq \chi_S \) and \( D = S = \frac{k}{k} \) \( (1 - \chi_S), \) and \( p_{1,B} = p_{1,S}^S \)

   (b) Or \( \chi_S \in \left[ \frac{k}{k}, \frac{k}{k} \right] \) and \( p_{1,B} = \frac{1}{k} \) \( \chi_S \) \( (1 - \chi_S) \) \( p_{1,T}^T \) \( p_{1,L}^T \) \( p_{1,L}^T \)

   (c) Or \( \chi_S = 1, D = S = 0, \) and \( 0 \leq p_{1,B} \leq p_{1,S}^S \). No assets are traded

3. Or \( \chi_S = 1, D = S = 0, \) and \( 0 \leq p_{1,B} \leq p_{1,S}^S \). No assets are traded

### 4.7 The allocation program

We now endogenize bankers’ choice to enter the \( T \)-or a \( S \)-banking sector by observing that bankers will choose whatever banking business is more profitable. They compare \textit{ex ante} value functions as of \( t = 0, \) and choose an allocation \( \chi_S \) such as to solve

\[
\max_{\chi_S \in (0;1)} \chi_S V_{0}^{S,B}(p_{1,B}) + (1 - \chi_S) V_{0}^{T,B}(p_{1,B})
\]

where \( p_{1,B} \) is a market price for capital goods at \( t = 1 \) in state \( B. \)
Let’s define:

\[ \Delta : \mathcal{P}_{1,B} \to \mathcal{V}_{0,H} (\mathcal{P}_{1,B}) - \mathcal{V}_{0,H} (\mathcal{P}_{1,B}) \]

Recalling that equilibrium market prices for capital goods at \( t = 1 \) in state \( B \) are to be found in \([0, \overline{p}_{1,H}]\), it is sufficient to study \( \Delta \) on this interval. On \([0, \overline{p}_{1,H}]\), \( \Delta \) is a continuous, strictly decreasing function.

As, \( \Delta(0) = +\infty \), it can cancel at most once on this interval. We end up with the following proposition

**Proposition 6 (Allocation program).** Defining \( S = \Delta^{-1}(0) \cap [0, \overline{p}_{1,H}] \), the allocation program solves as follows

1. If \( S = \emptyset \), \( \chi^S = 0 \)
2. Otherwise, denoting \( \mathcal{P}_{1}^* = \Delta^{-1}(0) \cap [0, \overline{p}_{1,H}] \), we have

\[ \chi^S = \begin{cases} 
0 & \text{if } \mathcal{P}_{1,B} \in [0; \mathcal{P}_{1}^*) \\
\mathcal{P}_{1,B}^* & \text{if } \mathcal{P}_{1,B} = \mathcal{P}_{1}^* \\
1 & \text{if } \mathcal{P}_{1,B} \in (\mathcal{P}_{1}^*; \overline{p}_{1,H}] 
\end{cases} \]

### 4.8 The game equilibria

Having detailed the different parts of our equilibrium, we can now focus on the equilibrium determination of our game. We have the following result, which follows from the above propositions.

**Proposition 7 (Equilibria description).** In all equilibria, bankers invest all their initial endowment in the bank they set up. The bank invests all the funds obtained from the bankers in the productive investment technology. At \( t = 1 \) in state \( G \) no assets are traded on the capital goods market, and banks roll over their time 0 short-term risk-free debt (\( \mathcal{P}_{1,G} \in [1; \overline{\delta}_{1/z}] \)). In each date and state, risk-less short term debt (if any) is sold at par to the households, and the households purchase all the short term debt sold to them by the bankers.

And:

1. Either \( \Delta(\overline{p}_{1}^*) < 0 \). In this case, there is a unique \( \mathcal{P}_{1,B}^* \in (0; \overline{p}_{1}^*) \) such that \( \Delta (\mathcal{P}_{1,B}^*) = 0 \). Then \( \chi^S = 1 \) is the unique equilibrium allocation, and any \( \mathcal{P}_{1,B} \in [\mathcal{P}_{1,B}^*; \overline{p}_{1,H}] \) is an equilibrium market price at \( t = 1 \) in state \( B \). No assets are traded in these equilibria, and only \( S \) banks exist. They don’t issue any form of risk-free debt.

2. Or \( \Delta(\overline{p}_{1}^*) = 0 \). In this case, any \( \chi^S \) such that \( \frac{\mathcal{P}_{1,B}^*}{\mathcal{P}_{1,B}^*} \leq \chi^S \) is an equilibrium allocation, and \( \mathcal{P}_{1,B} = \mathcal{P}_{1,B}^* \). Either only \( S \)-banks exist, and they don’t issue any type of risk-free debt, and invest all their endowment in the productive investment, or shadow and traditional banks coexist, and interact on the asset market: \( T \)-banks raise funds at \( t = 0 \) to invest in the productive investment technology, and at \( t = 1 \) in state \( B \), to purchase capital goods on the market.

3. Or \( \Delta(\overline{p}_{1}^*) > 0 \) and \( \Delta(\overline{p}_{1}^*) < 0 \). Then, there is a unique \( \mathcal{P}_{1,B}^* \in (\overline{p}_{1}^*, \overline{p}_{1,H}) \) such that \( \Delta (\mathcal{P}_{1,B}^*) = 0 \). In this case \( \chi^S = \frac{1}{1 + \frac{\mathcal{P}_{1,B}^*}{\mathcal{P}_{1,B}^*} \frac{\mathcal{P}_{1,B}^*}{\mathcal{P}_{1,B}^*} \frac{n - 1}{k}} \) is the unique equilibrium allocation, and \( \mathcal{P}_{1,B} = \mathcal{P}_{1,B}^* \) is the unique
equilibrium market price at $t = 1$ in state B. Shadow and traditional banks coexist, and interact on the asset market at $t = 1$ in state B: T-banks don’t issue short-term debt at $t = 0$, but issue short term debt at $t = 1$ in state B in order to purchase underpriced capital goods from S-banks. S-banks lever at $t = 0$, and sell capital goods on the market at $t = 1$ in state B.

4. Or $\Delta(p^T_1) = 0$. In this case, any $\chi^S$ such that

$$\frac{k}{p^T_1 qr} \frac{1}{1 - (1 - \epsilon)} + \frac{k}{p^T_1 qr} \geq \chi^S$$

is an equilibrium allocation, and $p^T_{1,B} = p^T_1$. Either only T-banks exist, and they issue $k$ units of risk-free debt at $t = 0$, or both S and T-banks exist, and T-banks issue less claims at $t = 0$, to keep slack to purchase capital goods at $t = 1$ in state B from S-banks. Shadow and traditional banks can coexist in this case, in which case assets are traded between the two types of intermediaries.

5. Or $\Delta(p^T_1) > 0$. In this case $\chi^S = 0$ and any $p^L_{1,B} \in \left[ p^T_{1,L,1} ; p^T_{1,H,1} \right]$ such that $\Delta(p^L_{1,B}) \geq 0$ is an equilibrium market price. No assets are traded in these equilibria. Only T-banks exist, and they issue $k$ units of risk-free debt at $t = 0$, which they roll over at $t = 1$ in state B and in state G.

The conclusion we draw from Proposition 7 is that both types of intermediaries can coexist and interact on capital goods market. When they do, S-banks lever themselves thanks to T-banks’ ability to purchase capital goods sold by shadow banks in a crisis (i.e. at $t = 1$ in state B). In such a situation, fire-sales always occur.

Indeed, T-banks’ have a limited ability to issue short-term debt, and thereby the total quantity of investment they can make at $t = 0$ and $t = 1$ in state B is limited. In order to purchase capital goods from S-banks in the crisis, T-banks need to be compensated for foregoing investment opportunities at $t = 0$. T-banks face a trade-off between issuing short-term debt at $t = 0$ and keeping some buffer in order to issue short-term debt at $t = 1$ in state B so as to purchase S-banks’ capital goods. Interestingly this makes the fire sale prices always lower than the price at which the traditional banks value the asset (i.e. $\delta$): the fire sale is not entirely driven by the need for T-banks to be compensated for higher functioning costs. However, the fire-sale price cannot be too low either: S-banks have to be be better off paying such a cost at $t = 1$ in state B to benefit from the intermediated government guarantee, in order to benefit from increased leverage in good times, rather than not leveraging up at all.

These trade-offs are key to understand the occurrence of fire-sales in a crisis and the interaction between traditional and shadow banks.

5 Discussion

5.1 The effects of assets transfer

Complementarity between T-banks and S-banks In the model, S-banks can issue risk-free claims by relying upon T-banks’ asset purchases in a crisis. T-banks are better off when there are more shadow banks because they have to pay a lower price for shadow banks’ assets in a crisis. In that sense, our model exhibits a form of complementarity between T-banks and S-banks that goes through the asset market at $t = 1$ in state B (henceforth “in a crisis”).
One implication of this complementarity is T-banks channel the support from the guarantee fund to the rest of the financial system. Indeed, T-banks have the possibility to intermediate this support by providing back-up to S-banks in times of troubles. In our setting, the more T-banks, the higher the (indirect) support from the guarantee fund to the financial system in a crisis. The higher this support, the better off the S-banks. Conversely, the more S-banks, the higher the amount of capital goods that needs to be absorbed by T-banks when S-banks need to delever. This induces a decrease in prices, as T-banks are limited in their ability to issue risk-free debt in a crisis. This also increases T-bank’s profit from asset purchases. T-banks are therefore better off when there are more S-banks.

The complementarity induced by the secondary asset market has consequences in terms of bankers’ allocation between the two types of banks, and the aggregate level of investment in the economy (hence welfare).

**Sectoral allocation effect and welfare impact** To clarify this point, let us conduct the following thought experiment. Take our model and assume that there is no secondary asset market in a crisis. In such a case, there cannot be asset trade in a crisis, which means that neither T-banks nor S-banks can use the proceeds of asset sales to repay their creditors. A notable difference between T- and S-banks is that T-banks have access to the guarantee fund making their debt safe in a crisis, while shadow banks do not. As a result, it is impossible for shadow banks to issue debt at \( t = 1 \) in state \( B \) and - backwardly - neither at \( t = 0 \), because there is a nonzero probability that S-banks’ output goes to zero at future date \( t = 2 \). In contrast, because they can access the guarantee fund, T-banks can issue risk-less debt in a crisis. From the viewpoint of \( t = 0 \), T-banks are able to issue risk-less debt insofar as they are able to refinance this debt at future dates, whatever state materializes at \( t = 1 \).

When there is no secondary asset market in a crisis, bankers’ choice to enter the T- or S-banking sector becomes the following. One the one hand, T-banks enjoy deposit insurance in a crisis. Even though they do not benefit from fire-sales of S-banks’ assets, this enables them to issue debt at \( t = 0 \). Again this comes at the cost of being regulated, which is captured by the parameter \( \delta < 1 \) in the model. On the other hand, S-banks do not have to comply with regulatory costs, and they do not enjoy deposit insurance in a crisis. Even without a secondary asset market, the absence of deposit insurance is detrimental to S-banks because they cannot issue debt at \( t = 0 \), thus no debt at all. Bankers therefore choose either to enter a levered though regulated T-banking sector, or an unregulated though unlevered S-banking sector.

Keeping the same notations as in the model, the time-0 value function of a banker entering the S-banking sector in an environment where there is no asset market in a crisis writes:

\[
V_{0}^{SB,NM} = [pR + (1 - p)qr]n
\]  

(8)

where we denote "NM" for "no market". Meanwhile, time-0 value function of a banker entering the T-banking sector when there is no asset market in a crisis writes:

\[
V_{0}^{TB,NM} = p\delta[R(n + k) - k] + (1 - p)(\delta qr(n + k) - k)
\]  

(9)
This has two implications. First, it should be noted that these value functions are a lower bound for the value function of bankers entering the T- and S-banking sectors in the model with an asset market. This implies that opening up the asset market can only generate gains from trade, hence can only positively impact the welfare of the economy. One example of such a welfare improvement associated to the existence of this market is detailed in Figure 7.

Moreover, in a general way, the existence of such an asset market impacts the allocation of bankers in one type of bank or the other. In Figure 7, we provide an example where T-banking is dominated when markets are closed ($V_{TB, NM}^0 < V_{SB, NM}^0$). Once we open up the asset market, bankers start allocating themselves in the T-bank sector. We interpret this as a potential rationale for why T-banks continue existing, even though their business model is dominated absent the opportunity to earn a profit from fire sales, the latter stemming from their ability to benefit from crises by issuing deposits when other intermediaries cannot.\footnote{The allocation of bankers between the two forms of intermediation technology is illustrated in Figure 8, both with and without asset market, for different values of $k$.} The shaded area represents the welfare gains from trade on the asset market at $t = 1$ in state B.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Value functions against asset price in a crisis ($V_{T}^0$ in red, $V_{S}^0$ in black, NM dashed)}
\end{figure}

5.2 Impact of changes in the level of guarantees ($k$) on the relative size of the traditional and shadow banking sectors

We now turn to the analysis of comparative statics of the model. In so doing, we focus on equilibria of type 3, as described in point 3 of proposition 7.

In an equilibrium of type 3, recall that bankers allocate themselves between T-banks and S-banks such that the value function at $t = 0$ of entering the T- or S-banking sector is the same (the solution is
interior). We have \( V_0^T = V_0^S \) with

\[
V_0^T = \left[ p (\delta R) + (1 - p) \left( \delta q r + \frac{\delta - p_{1,B}^*}{p_{1,B}^*} k \right) \right] n
\]

\[
V_0^S = \left[ p \left[ R - \frac{\alpha}{1 - \epsilon} q r \right] \right] n
\]

where \( p_{1,B}^* \) is the market equilibrium price for capital goods at \( t = 1 \) in state \( B \).

Denoting \( \chi^S \in [0, 1] \) the equilibrium share of S-banks, market clearing condition in a crisis yields:

\[
p_{1,B}^* = \frac{k}{(1 - \epsilon) q r \left( \frac{\chi^S}{1 - \chi^S} + \frac{k}{n} \right)}
\]

Recall that for parameter values such that this equilibrium exists, it is uniquely defined. Combining these three conditions we obtain

\[
V_0^S = n p \left( \frac{R - 1}{\chi^S - 1} \right) \left( \frac{\chi^S}{1 - \chi^S} + \frac{k}{n} \right) + np \tag{10}
\]

\[
V_0^T = np \delta R + (1 - p) \left( \delta q r n + (\delta (1 - \epsilon) q r - 1) k + \delta (1 - \epsilon) q r \frac{\chi^S}{1 - \chi^S} n \right) \tag{11}
\]

which enables us to link the equilibrium sectoral allocation to the value function of T-banks and S-banks.

We use expressions (10) and (11) to discuss how the relative size of the two banking sectors is modified when allowing for a slight change in the size of the government guarantee.

**Change in the amount of deposit insurance** \( k \)  
For clarity we discuss the effects of lowering \( k \). We first focus on a change in \( k \) low enough to ensure that the considered modified equilibrium stays of type 3. Two competing effects are at stake.

1. On the one hand, keeping all parameters constant, a lower \( k \) reduces the size of the advantage of being a T-bank: T-banks have access to a lower government guarantee, which limits their leverage at all times, hence exerting a downward pressure on T-bank’s profits. This makes T-banking relatively less profitable than S-banking, thereby inducing bankers to enter the S-banking sector. We call this direct effect a substitution effect.

2. On the other hand, S-banks indirectly benefit from T-banks’ guarantee fund through the (secondary) asset market in a crisis: this is at the root of the complementarity effect we pointed out in the previous subsection. Lowering \( k \) reduces the support that can be provided to S-banks through T-banks, reducing the price at which capital goods can be sold on the secondary market, hence reducing S-banks’ profits and the incentive of entering the S-banking sector. We call this effect an income effect.

\[\text{It is reminiscent of the argument at play in existing models of shadow banking as regulatory arbitrage (see e.g. Plantin (2015), Ordonez (2013), Harris et al. (2015)).}\]
In our setting, the asset price adjustment associated to a lower asset demand by T-banks in a crisis outweighs the direct effect of lowering \( k \) on T-banks’ profits: the net effect of lowering \( k \) on T-banks’ expected profits is positive taking bankers’ allocation as given.\(^{27}\) We find that lower support to T-banks in a crisis reduces asset prices to such an extent that more bankers choose to enter the T-banking sector \textit{ex-ante}, i.e. the income effect outweighs the substitution one. Bankers therefore react by allocating themselves with a higher probability in the traditional banking sector.

However, this does not hold true for high changes in \( k \): holding \( \delta \) fixed, when \( k \) gets low enough, T-banking becomes too inefficient for T-banks to continue existing. Indeed, for low levels of \( k \), the T-banks would require too high a compensation for purchasing the asset at \( t = 1 \) in state \( B \) that S-banks would be better off not issuing short term debt. Moreover, in this range, entering the unlevered shadow banking sector is more valuable than entering the traditional banking sector where banks issue \( k \) units of risk-less claims at \( t = 0 \) to invest in the productive technology. Finally, bankers allocate themselves only in S-banking, and no short-term debt is issued.

Figure 8 illustrates the allocation of bankers into the shadow banking system when the parameters are such that S-banking is dominating absent a market for assets in a crisis (i.e., at \( t = 1 \) in state \( B \)). We take \( n = 1, p = 0.9, r = 1, q = 0.99, R = 1.13, \varepsilon = 0.11, \delta = 0.9 \), for \( 0 < k < k^\ast \). The dashed red line illustrates the fact that all bankers allocate to the S-banking sector if T-banks cannot purchase capital goods from S-banks in the secondary market. The plain black line illustrates the two effects at play for different values of \( 0 < k < k^\ast \).\(^{28}\) When \( k \) is low, T-banks’ advantage of issuing debt in a crisis do not compensate for their costs, and all bankers allocate to the S-banking sector (\( \chi^S = 1 \)). Once T-banks’ advantage enables them to purchase assets from S-banks at fire-sale prices, not all bankers allocate to the S-banking sector and T- and S-banks coexist. By the same reasoning as previously, increases in the amount of deposits guaranteed in a crisis pushes more bankers to allocate to the S-banking sector.

\(^{27}\)This is evidenced by the fact that, in equation (10), the coefficient of \( k \), \( (1 - p) (\delta (1 - \varepsilon) q r - 1) \frac{1}{n} \) is negative.

\(^{28}\)Notice that, for these values of the parameters, coexistence of the different forms of intermediation is ensured only when asset markets are open.
Analogy with capital requirements  A positive analogy that can be developed is to consider a model where the structural limit on the size of T-banks’ support in a crisis \((k)\) comes at the cost of complying with capital requirements, that are imposed to T-banks for reasons outside of the model.

We assume that at \(t = 1\) in state \(B\), T-banks face a capital requirement of the form:

\[
D_{1,B}^T \leq (1 - c) \left[ \alpha l_0^T + p_{l,B} qr l_1^T \right]
\]

(12)

where \(D_{1,B}^T\) denotes the amount of funds borrowed from households and \(p_{l,B}\) the price of capital goods.\(^{29}\) Such a model would yield similar results, where our \(k\) counterpart would be found in \((1 - c)\). As is the case in our model, imposing such a capital requirement on deposits issued in a crisis spills over to date 0, as T-banks needs to roll all their debt over at \(t = 1\) in state \(B\), when no other intermediary is able to purchase assets. The same argument as in our model can then be adapted to this case, where increased capital requirement in a crisis (thereby at all dates and states) would reduce T-bank’s ability to purchase assets in a crisis, making T-banks better off and reducing bankers’ incentives to set up a S-bank \textit{ex-ante}. Conversely, lower capital requirement in a crisis would reduce the impact of fire sales: In that way, policies such as countercyclical capital buffers could have the potentially unwanted consequences of increasing the support to the shadow banking system at the time when shadow banks need it the most, hence driving more intermediaries into setting up unregulated entities.

Post-crisis banking reforms  There have been several policy initiatives to impose restrictions on banks’ trading activities since the crisis. Prohibiting regulatory arbitrage is the paradigm in Section 619 of the

\[^{29}\text{At } t = 1 \text{ in state } B, \text{ we would set the following constraint, for the capital constraint to be stringent enough (rhs) while not incitating T-banks to delever at } t = 1 \text{ in state } B \text{ (lhs):} \]

\[
\delta (1 - \epsilon) qr \leq (1 - c) \leq \delta qr
\]

(13)

The philosophy of these reforms is to prevent traditional banks from doing regulatory arbitrage by sustaining shadow banking activities, which Pozsar et al. (2013) refer to as “internal shadow banking”. Through the lens of our model, we indeed acknowledge the fact that contractual relationships on liquidity guarantees from traditional to shadow banks tend to favor shadow banking by increasing asset prices in a crisis. However, our key finding is that, even absent contractual relationships between traditional and shadow banks, the two types of banks coexist because they are not only substitutes but also complements. The complementarity between the two bank types comes from shadow banks’ asset (fire) sales to traditional banks in a crisis. One implication of this complementarity is that traditional banks channel the support from the guarantee fund to shadow banks, even absent a contractual relationship between the two. Although it might then seem like a good idea to prevent traditional banks from trading assets in a crisis, this paper argues that traditional banks’ profits from shadow banks’ fire-sold assets in a crisis outweigh the (regulatory) costs that traditional banks have to comply with. This explains why traditional banks still exist even though shadow banks perform comparable activities while not having to comply with regulations. When designing banking reforms one needs to think hard about the implications of reforms such as a shutdown of the (interbank) asset market, in light of the reasons why regulated and unregulated banks coexist in the first place. This paper provides a framework to do so.

5.3 Normative approach

In addition to the positive aspects developed above, this framework provides a setting for a normative analysis. To do so, we will look at the aggregate surplus in the economy, hence the aggregate profit of the traditional and shadow banking systems. A way to think about it is to consider that bankers don’t consume the profits they make, but that they provide the households with it in the end of period 2. This point of view is close to the normative measure put forward by Stein (2012).

Among the questions that can be addressed, we focus here on the analysis of the efficiency of the banker’s allocation between the two types of banking institutions, with respect to the measure above. We will be assuming that having fixed the allocations of bankers between the two forms of intermediation technology, they are free to make the investment decisions that occur in the decentralized market.

In this model, the advantage of shadow banks is that they don’t have to bear the cost of regulation $\delta$ (and the cost of early investment disruption $\epsilon$), while the advantage of traditional banks is that they can get support from the government in times of crises $k$. These two distinctions might lead to some form of inefficiencies, associated to pecuniary externalities linked to a misallocation of agents.

Let’s now consider the allocation problem faced by the central planner. The problem of the central plan-
ner writes as the allocation program of the agents. However, he recognizes the impact of its allocation choice on the equilibrium price \( p_{1,B} \) which clears the secondary market in time 1, B. The planner chooses an allocation \( \chi^S \in [0;1] \) such as to solve the central planner’s program:

\[
\max_{\chi^S \in [0;1]} \chi^S V^S_0 (p_{1,B}) + (1 - \chi^S) V^T_0 (p_{1,B})
\]

where \( p_{1,B} \) is a market price for capital goods at \( t = 1 \) in state B. \( \chi^S \) being fixed, shadow and traditional banks make the same choices as in the decentralized equilibrium and \( p_{1,B} \) is expressed as a function of \( \chi^S \) in the same way as in Proposition 5.

The objective function of the central planner can then be rewritten as a piecewise linear function of \( \chi^S \):

\[
W(\chi^S) = \begin{cases} 
\chi^S V^S_0(\frac{p_{1,L}^T}{p_{1,L}^T}) + (1 - \chi^S) V^T_0(\frac{p_{1,L}^T}{p_{1,L}^T}) & \text{if } \chi^S \in [0;\chi^S] \\
\alpha_0 + \alpha_1 \chi^S & \text{if } \chi^S \in [\chi^S;\chi_S] \\
\chi^S V^S_0(\frac{p_{1,T}}{p_{1,T}}) + (1 - \chi^S) V^T_0(\frac{p_{1,T}}{p_{1,T}}) & \text{if } \chi^S \in [\chi_S;1] 
\end{cases}
\]

with \( \alpha_0 = p [R - 1] + n p R + (1 - p) [\delta (1 - \epsilon) q r - 1] \), \( \alpha_1 = n [p R + (1 - p) \epsilon q r] + k (1 - [p R + (1 - p) \delta (1 - \epsilon) q r]) \), \( \chi^S = \frac{(n + k)(1 - \epsilon q r - \epsilon q r)}{1 - (1 - \epsilon q r) - \epsilon q r} \).

For the following, we restrict our attention on parametric restrictions where the decentralized equilibrium is such that both traditional and shadow banks coexist and the market price is such that \( p_{1,B}^D \in (p_{1,T}^S, p_{1,L}^T) \). This corresponds to an equilibrium of type 3 in the typology of Proposition 7. In this case, the banker’s allocate themselves with a uniquely defined probability \( \chi^D_S \in [\chi^S;\chi_S] \) into the shadow banking system.

Such parameter restrictions first ensure that \( W(.) \) is strictly increasing on \([0;\chi^S]\) and strictly decreasing on \([\chi^S;1]\). Moreover, from \( V^T_0(\frac{p_{1,T}}{p_{1,T}}) > V^S_0(\frac{p_{1,T}}{p_{1,T}}) \), we can infer that \( \alpha_1 < 0 \) 30 In turn, \( W(.) \) is strictly decreasing on \([\chi^S;\chi_S]\) and the constrained optimum allocation is uniquely obtained for \( \chi^S_C = \chi^S < \chi^D_S \). The market allocation of bankers into the shadow banking sector is in this case always excessive. We illustrate this situation in figure 9, where the parameter values used in figure 7 have been chosen.

30 Indeed, \( V^T_0(\frac{p_{1,T}}{p_{1,T}}) > V^S_0(\frac{p_{1,T}}{p_{1,T}}) \) rewrites \( [p R + (1 - p) \epsilon q r] [1 - \delta] n < k (1 - \epsilon) (p R - 1) + (1 - p) \epsilon q r - (1 - p)] \) which implies \( n [p R (1 - \delta) - \delta (1 - p) \epsilon q r] < k (p R + (1 - p) \delta (1 - \epsilon) q r - 1) \) or alternatively \( \alpha_1 < 0 \).
With these parameters’ sets, bankers allocate themselves too much into the shadow banking system, or conversely too little into the traditional banking system. The key mechanism at play in this inefficiency is reminiscent of Stein (2012), and stems from pecuniary externalities; bankers, by choosing to allocate themselves into the shadow banking system fail to internalize that this will lower the support brought to shadow banks in bad times. This in turns hinders the ability of traditional banks to provide a backstop in times of crises, hence the ability of all shadow banks in growing and investing in normal times; in a nutshell, when bankers allocate themselves into the shadow banking system, they fail to take into account that this choice reduces the ability of traditional banks to purchase assets in bad times, hence the ability of all shadow banks to issue risk-free claims in time 0.

In this case, transfers between the two forms of intermediation technology can be beneficial in terms of total Welfare. The aim is here to incentivize more agents to allocate themselves towards the traditional banking system, by subsidizing traditional banking activities and financing this subsidy through taxes levied on shadow banks. Welfare can always be improved by imposing lump sum taxes on shadow banks, in order to subsidize traditional banking activities.

It should be noted that the market allocation needs not be always inefficient. For instance, if the decentralized allocation is one in such shadow banks are the only form of intermediaries existing, decentralized and centralized allocations will coincide, as exemplified in the following figure.
6 Conclusion

We document and integrate stylized facts from the 2007 financial crisis into a simple model that rationalizes the coexistence of traditional and shadow banks. This paper offers the first model of financial intermediation where both a regulated and an unregulated sector coexist and interact, while replicating the following facts from the crisis: (i) liabilities transfer from shadow to traditional banks, (ii) assets transfer from shadow to traditional banks, and (iii) fire sales of assets.

The model describes the different technologies used by traditional and shadow banks in order to issue safe claims against risky collateral. On the one hand, traditional banks rely on a guarantee fund to be able to issue risk-free claims in a crisis. Therefore this guarantee also enables them to issue risk-free claims outside a crisis. This access to a guarantee fund comes at a cost of higher regulation. On the other hand, shadow banks rely upon traditional banks’ ability to issue risk-free claims in a crisis, to absorb their assets and provide them with enough liquidity to repay their debt holders. This interaction in the asset market enables traditional banks to intermediate government insurance to the rest of the financial system, generating a complementarity between the two forms of financial intermediation. This complementarity is the key message of this paper: the more shadow banks in the system, the lower the price traditional banks have to pay for shadow banks assets in a crisis, and the better off the traditional banks. We see this form of complementarity as a main driver of the coexistence of these two banking sectors in the financial system.

We find that when shadow and traditional banks coexist in an economy, a small reduction in traditional banks’ ability to issue deposits in a crisis induces a shift of intermediation towards the traditional banking sector. Indeed, two opposite effects are at play. One is the direct effect of hindered traditional banks’ ability to raise funds in a crisis, which is to reduce their ability to lever up and thereby their expected profits. This substitution effect induces bankers to shift to the shadow banking sector, as this latter is not directly impacted by such a reduction in traditional banks ability to issue risk-less debt.
The other one is indirect and akin to an income effect: a reduction in traditional banks’ ability to issue risk-free claims induces a lower asset demand in the crisis, creating a downward pressure on the equilibrium price of assets transferred from the shadow to the traditional banking sector. This lowers shadow banks’ expected profits and increases traditional banks’ expected profit, thereby pushing more bankers into the regulated sector. We show that this latter effect is sizeable and outweigh the former. All in all, a reduction in traditional banks’ ability to lever up in a crisis, if not too strong, leads to an increase in the relative size of the traditional banking sector.

From a normative perspective, we show that, when traditional and shadow banks coexist, the pecuniary externality stemming from the fire sales of shadow banks assets in times of crises usually induces inefficiencies: bankers tend to allocate too much into the shadow banking system, failing to internalize the fact that their allocation towards the shadow banking system reduces the overall support that can be brought to the shadow banking system in times of crises. Lump sum transfers between shadow and traditional banks in an attempt to increase the number of traditional banks have an overall positive impact on the welfare of the economy.
References


European Systemic Risk Board ESRB. Eu shadow banking eu shadow banking monitor. *ESRB Reports*, 1, July 2016.


A The data

A.1 Stylized balance sheets of US financial intermediaries

We aggregate those financial intermediaries that we include in our definition of the shadow banking sector\textsuperscript{31}, we define short-term debt using the FAUS by using the same terminology as Krishnamurthy and Vissing-Jorgensen (2015). We build stylized balance-sheets by consolidation of the financial balance sheets of the legal institutions for which we have data in the Financial Accounts of the United States (FAUS).

The list of FAUS items included in shadow banks’ short-term debt is: Security repurchase agreements (net), Depository institution loans n.e.c., Trade payables, Security credit (Customer credit balances), Security credit (U.S.-chartered institutions), Security credit (foreign banking offices in U.S.), Taxes payable, Commercial paper, Open market paper.

A.2 Fact 1: Liabilities flow from shadow to traditional banks

A.2.1 Table 1

We take the definition of the largest US bank-holding companies on Figure 11 from the Federal Reserve’s website (https://www.ffiec.gov/nicpubweb/nicweb/HCSGreaterThan10B.aspx/).

<table>
<thead>
<tr>
<th>Cumulative flows since 2006q4</th>
</tr>
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<tbody>
<tr>
<td>Shadow banks ($Bill)</td>
</tr>
<tr>
<td>2007q2</td>
</tr>
<tr>
<td>2007q3</td>
</tr>
<tr>
<td>2007q4</td>
</tr>
<tr>
<td>2008q1</td>
</tr>
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<td>2008q3</td>
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<td>2008q4</td>
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<td>2009q3</td>
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</tr>
<tr>
<td>2010q4</td>
</tr>
<tr>
<td>2011q1</td>
</tr>
<tr>
<td>2011q2</td>
</tr>
</tbody>
</table>

Table 1: Traditional and shadow banks: short-term debt inflows (negative values denote outflows) source: Financial Accounts of the United States. We define traditional, shadow banks, and short-term debt in Appendix A.1.

\textsuperscript{31}Recall we define shadow banks as the sum of .
A.2.2 Figure 11

Figure 11: Large traditional banks: deposits and borrowings (stocks in $ bn)

source: Fed H8 Releases

A.2.3 Book versus market value of equity

He et al. (2010) and Bigio et al. (2016) also find that traditional banks’ book equity increased by around US $250 billion during the crisis. Figure 12 provides evidence of this increase in the stock of book equity of the US traditional banking sector through the crisis. This Figure is based on reported book value of equity, which is the leverage measure most used for regulatory purposes. However, there are reasons to believe that the true level of capital for the traditional banking sector was lower. We use data from CRSP to measure the market value of traditional banks’ equity and we see that most of the increase in
A.2.4 Fact 2: Asset flow from shadow to traditional banks

A.3 Regression: Traditional banks’ MBS purchases in the crisis

One main testable prediction of our theory is that traditional banks are able to purchase assets from shadow banks in a crisis, insofar as they benefit from a guarantee on their deposits. This guarantee indeed enables them to attract deposits precisely when shadow banks have to repay their creditors. Publicly available data on purchases/sales of assets by traditional and shadow banks during the crisis is not available. Therefore we try to estimate purchases/sales of mortgage-backed securities (henceforth MBS) applying He et al. (2010)’s methodology on traditional banks’ regulatory data from the Call Reports. We observe the total value of MBS holdings by each traditional bank before the crisis (denote it $P_{2007q4} MBS_{2007q4}$, where $P_{2007q4}$ is the fair price of MBS securities in 2007q4 and $MBS_{2007q4}$ is the quantity of MBS held by bank $i$ in 2007q4) and after the crisis ($P_{2009q1} MBS_{2009q1}$). Besides, denoting $f$ the repayment/maturity rate of MBS net of the new issuance rate during the period from 2007q4 to 2009q1, the International Financial Reporting Standards (IFRS) give us the following accounting identity:

$$P_{2009q1} MBS_{2009q1} - P_{2007q4} MBS_{2007q4} \times (1 - f) = MBSPurchases_{i} - MBSLosses_{i}$$

As in He et al. (2010), we test three different scenarios based on (i) the total losses that traditional banks incurred on MBS assets during the 2008 crisis, and (ii) Bloomberg WDCI estimates for the net repayment rate $f$. Under scenario 1 the repayment rate used to construct the MBSPurchases variable is 7% and total losses imputed to the financial sector are $500$ billion.\(^{32}\) Under scenario 2, the repayment

\(^{32}\)Note that the only available estimate on MBS losses in the crisis is an aggregate over the traditional banking sector from

\[\text{Figure 13: Traditional and shadow banks: asset inflows (negative values denote outflows) source: Financial Account of the United States}\]
<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Naive Scenario</th>
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<td>0.227***</td>
<td>0.223***</td>
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<td>(0.044)</td>
<td>(0.039)</td>
<td></td>
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<td>−0.583***</td>
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<td>(0.117)</td>
<td>(0.109)</td>
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<tr>
<td><strong>Change_Net_Wholesale_fund</strong></td>
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<td>0.116***</td>
<td>0.113***</td>
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<td>(0.037)</td>
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<td>(0.049)</td>
<td>(0.041)</td>
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<td><strong>Change_NPL_Ratio</strong></td>
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<td>−0.287***</td>
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<td>(0.078)</td>
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<td>(0.031)</td>
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<td><strong>adjusted R²</strong></td>
<td>0.1989</td>
<td>0.2229</td>
<td>0.2374</td>
</tr>
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</table>

Source: Call Reports

*p < 0.10, ** p < 0.05, *** p < 0.01
rate is 12% and total losses are $176 billion. Under the "naive" scenario, we do not correct for the net repayment rate nor total losses.

We analyze the data formally by running the following OLS regression on changes in various items of traditional banks’ balance sheets from 2007q4 to 2009q1:

\[
MBSPurchases_i = \beta_0 + \beta_1.Liquidity_i + \beta_2.Solvency_i + \beta_3.Insured._deposits_i \\
+ \beta_4.Non._insured._deposits_i + \beta_5.Credit._granted_i \\
+ \beta_6.Evolutions_i + \beta_7.controls_i + \epsilon_i
\]

where \(MBSPurchases_i\) is our estimated purchases/sales of mortgage-backed securities by traditional bank \(i\) normalized by total assets (banks are aggregated to the top holder level in the Call Reports).

The data come from the quarterly Call Reports and He et al. (2010)'s estimates. We use the procedure described in Acharya and Mora (2015) to construct our sample. Variables ending in 2007q4 represent variable levels as of 2007q4. Variables starting with "Change" are growth rates from 2007q4 to 2009q1, normalized by total assets as of 2007q4. The dependent variable MBS_Purchases represents purchases of mortgage-backed securities by traditional banks between 2007q4 and 2009q1, normalized by total assets as of 2007q4. As in He et al. (2010) we test many scenarios in terms of MBS repayment rate and total losses on assets, to make sure that what our dependent variables capture are actual purchases of MBS by traditional banks. We report three of these scenarios, including the "naive" one. Under scenario 1 the repayment rate used to construct the MBS_Purchases variable is 7% and total losses imputed to the financial sector are $500 billion. Under scenario 2, the repayment rate is 12% and total losses are $176 billion. Under the "naive" scenario, we do not correct for the net repayment rate nor total losses. The White robust standard error estimator is used. Table 14 below details the construction of variables (mainly following Acharya and Mora, 2015).

The IMF’s Global Financial Stability Report of October 2008 and Bloomberg WDCI (which explains why we test two scenarios thereafter). Denote those estimates for the entire traditional banking sector losses on MBS assets MBSLosses. Although we try to estimate MBS purchases/losses by taking into account potential losses on those assets when using the change in MBS holdings from 2007q4 to 2009q1 adjusted for the net repayment/maturity rate, we cannot account for differences in losses across traditional banks. We therefore assume that losses incurred by traditional banks are proportional to the amount of MBS they hold, so that \(MBSLosses_i = \frac{MBS_i.2007q4}{\sum_k MBS_i.2007q4} \times MBSLosses\) and \(\sum_k MBSLosses_k = MBSLosses\).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Name</th>
<th>Call Report Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insured deposits</td>
<td>insured_deposits</td>
<td>rconf049 + rconf045</td>
</tr>
<tr>
<td>Brokered deposits</td>
<td>brokered_deposits</td>
<td>rcon2365</td>
</tr>
<tr>
<td>Interest rate on large deposits</td>
<td>ir_large_deposits</td>
<td>rconf049 + rcon045</td>
</tr>
<tr>
<td>Unused commitments</td>
<td>Credit</td>
<td>rcfdd0418 + rcfdd1817 + rcfdd6550 + rcon0411</td>
</tr>
<tr>
<td>Unused commitments ratio</td>
<td>Unused_commissions_ratio</td>
<td>unused commitments/(unused commitments + rcfdd1400)</td>
</tr>
<tr>
<td>Cash</td>
<td></td>
<td>rcfdd010</td>
</tr>
<tr>
<td>MBS</td>
<td></td>
<td>rcfdd350 + rcon0487</td>
</tr>
<tr>
<td>Securities (MBS excluded)</td>
<td>Liquid assets</td>
<td>Liquid Assets/rcfd2170</td>
</tr>
<tr>
<td>Wholesale funding ratio</td>
<td>Liquidity ratio</td>
<td>rconf2604 + rcfdb2000 + rcfdd3200 + rcon0993 + rcfdb995 + rcfdd3190</td>
</tr>
<tr>
<td>Net Wholesale fund ratio</td>
<td>Net_Wholesale_fund</td>
<td>Wholesale funding/(Securities (MBS excluded) + Federal Funds Sold + Cash)</td>
</tr>
<tr>
<td>Non performing loan</td>
<td>NPL @_ratio</td>
<td>Non performing loan/rcfd2140</td>
</tr>
<tr>
<td>Non performing loan ratio</td>
<td>Capital ratio</td>
<td>(rcfd2140/rconf3808)/rcfd2170</td>
</tr>
<tr>
<td>Real Estate Loan Share</td>
<td>Real_Estate</td>
<td>rconf0410 + rcfdd6000</td>
</tr>
<tr>
<td>Residential Mortgages</td>
<td></td>
<td>rcfdd0870 + rcfdd0701 + rcfdd2170</td>
</tr>
<tr>
<td>Financial Assets</td>
<td></td>
<td>rcfdd0081 + rcfdd0071 + rcfdd570 + rcon054 + rconf056 + rconf056 + rcon056</td>
</tr>
<tr>
<td>Short Term Liabilities</td>
<td>Maturity Gap</td>
<td>(Financial Assets - Short Term Liabilities) / rcfd2170</td>
</tr>
<tr>
<td>Mat_Gap</td>
<td></td>
<td>rconf167</td>
</tr>
<tr>
<td>Tag deposits</td>
<td>Tag_deposits</td>
<td>rcon049</td>
</tr>
</tbody>
</table>

Figure 14: Variables definitions

Note: All missing observations are considered equal to zero. Banks are aggregated to top holder level (RSSD9348). We follow the same procedure as Acharya and Mora (2015).

A.3.1 Fact 3: Asset fire sales

B The model

See online technical appendix.